1ST INSO

International Nuclear Science Olympiad
PHILIPPINES 2024

31 July to 7 August 2024
Pampanga, Philippines

Examination

Experiment Questions

General Instructions

Points: 20

Time: 3.5 Hours

General Instructions

Failure to comply with any of the following instructions may lead to disqualification.

The exam is worth a total of 20 points and will last for 3.5 hours.

The start and end times of the examination will be announced, hourly announcements will be made, and a reminder will be provided when there are 15 minutes remaining.

Do not open the exam envelopes until instructed to do so.

The following items are provided on your table:

- Kit (1 ballpen, 1 pencil, 1 eraser, 1 ruler, 1 scientific calculator)
- 1 laptop with mouse and charger
- Snacks and water bottle
- Red envelope for answer sheets
- 5 signal cards:
  - Signal for additional water bottles, papers, or pens: "**KIT**"
  - Signal for toilet break: "**TOILET**"
  - Signal for clarification: "**?**"
  - Signal for medical aid: "**MEDIC**"
  - Signal for PC error or program error: "**ERROR**"

The location of items on your table are shown in the figure below:

During the exam:

- Use the ballpen provided. If you use the pencil to draft your notes, figures, tables, and graphs, make sure to trace the outlines of the final version with the ballpen.
- Use the software and computer applications installed in the laptops to answer the experiment questions. Raise the "Error" signal in case you need assistance related to the laptop.
- Use the Answer Sheets for your final answers. Fill in appropriate sections with your answers. Draw graphs as required. Cross out any unneeded answers.
- Blank working sheets are provided. Additional sheets are available upon request. Raise the "KIT" signal to notify the proctors.
- Keep your answers concise and legible. Use equations, operators, symbols, and sketches to convey your thoughts effectively.
- Uncertainty quantification is not required unless specified otherwise.
- Write numerical answers with 4 significant figures unless specified otherwise or when limited by the number of significant figures given in the problem. Use proper scientific notation as necessary, particularly for small (<0.01) and large numbers (>999).
- Do not leave your booth without permission. If you need a washroom break or other assistance, raise the appropriate signal(s) marked "Toilet", "Kit", "?", "Medic", or "Error".

At the end of the exam:

- Stop writing immediately when the end of the exam is announced.
- Place all answer sheets in the red envelope and seal. Sign across the envelope flap. Your proctor will assist you in securing the sheets in the envelope and will collect them afterwards.
- Place all other sheets (question paper, scratch paper, etc.) in their original envelope and leave on the table.
- Leave the exam kit on your table. You may take the remaining items with you, for example, the bottle of drinking water, and snacks.
- Your Proctor will let you know when you can leave.

Points: 20

Time: 3.5 Hours

Q1 ANALYSIS OF SEDIMENTATION PATTERNS USING LEAD-210 (10 pts)

In this experiment, alpha spectrometry data from sediment cores obtained from Sorsogon Bay, will be used to obtain the sedimentation rate in the region by calculating the concentration of Lead-210 (Pb-210) in the samples and validating the results using the Cesium-137 (Cs-137) dating technique. An Excel template is provided to aid you in performing the data analysis for the experiment. Each part of this question has a corresponding Excel sheet tab as shown below:

(1.1) Decay Constants | (1.2) Tracer Activity | (1.3) Relative Efficiency | (2.1) Total Pb-210 | (2.2) Pb-210 Plot | (4.1) Cs-137 Plot | (4.2) t-tab values

Instructions for filling up the Excel file is enumerated throughout the problem in alphabetical bullets (e.g. a. In **"Tab (1.1) Decay Constants"**, calculate the decay constant). This is important as the Excel calculations will help in answering the subsequent exam questions.

Write numerical answers with only 4 decimal places (rounding up) in the answer sheet. However, use the exact value (without rounding off) in Excel when doing calculations. Refer to calculated tables and reference cells when necessary. All numerical answers should be written with appropriate SI units.

Experiment Guide

The Pb-210 dating technique is a well-established method in geochronology and environmental science. It can be used to determine the age of sedimentary deposits and the rate of sedimentation in lakes, oceans, and other bodies of water, to study sedimentation patterns, environmental changes, erosion processes, and transport of nutrition and pollution. Pb-210 (half-life = 22.3 years) is a member of the U-238 decay series which accumulates in the environment in two pathways (**Figure 1**).

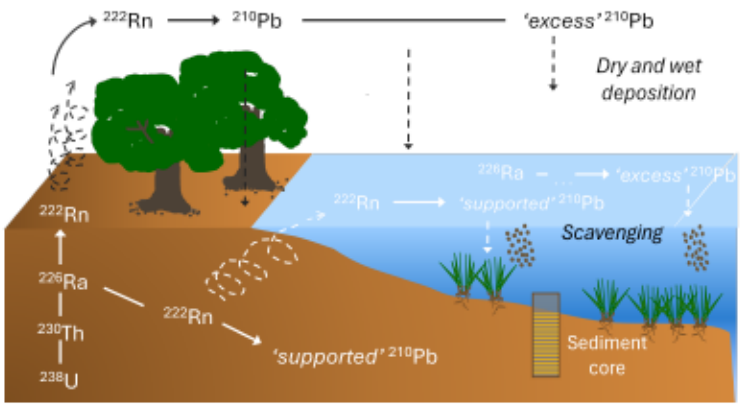


Figure 1. *The Pb-210 transport pathway*

The atmospheric origin for Pb-210 (known as *unsupported* or *'excess' Pb-210)* involves the decay of radioactive gas radon, Rn-222, which is produced in the earth and escapes to the atmosphere. Rn-222 has a half-life of 3.8 days and undergoes decay through four daughter products with short half-lives making its decay to solid Pb-210 geologically instantaneous. The Pb-210 settles out of the atmosphere or scavenged by rainwater and is then bound by organic matter and deposited in an environment such as oceans, lakes or soil. Once isolated in a sediment, Pb-210 is assumed to be immobile.

Meanwhile, the terrestrial origin of Pb-210 involves the in-situ decay of particulate Ra-226 and its daughter Rn-222 rising from sediments and rocks at depth (termed as *'supported' Pb-210)*. The *'Supported' Pb-210* is in secular equilibrium with its parent radionuclide, meaning it is continuously produced by the decay of Ra-226 at a rate that balances its own decay. The Pb-210 that can be measured in sediment samples is called the total Pb-210 (*Total Pb-210 = 'supported' Pb-210 + 'excess' Pb-210*). It is essential that the total Pb-210 within a sediment sample be separated into these two components. Apart from the surface, which is a layer prone to mixing or bioturbation, i.e. restructuring of sedimentary deposits by animals or plants, it is assumed that the 'excess' Pb-210, once incorporated in the sediment, decays exponentially with time as seen in **Figure 2**. Thus, the 'excess' Pb-210 is the only component relevant to *sedimentation rate*, or the rate at which soil/sediment accumulates over time (thickness/time).

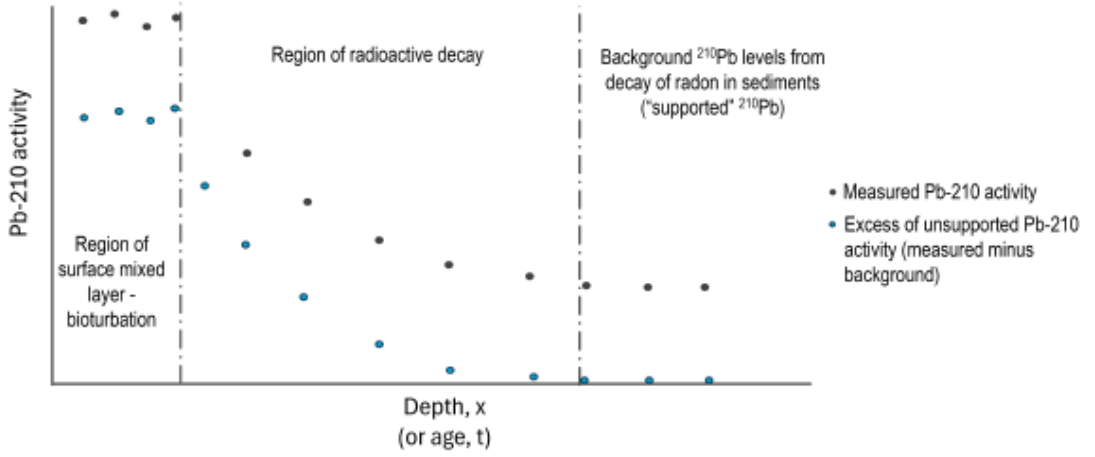


Figure 2. *Ideal profile of Pb-210 activity in sediment cores*

Experiment Scenario

In this study, sediment cores have been collected from Sorsogon Bay, to investigate and document the sedimentation patterns using Pb-210. The general steps are summarized in **Figure 3**. The Pb-210 can be determined by measurement of its daughter nuclide, Po-210 (half-life = 138.4 d), which decays 100% by alpha-particle emission. Pb-210 decays to Po-210 100% by beta-particle emission. The activity can be calculated by assuming secular equilibrium between Pb-210 and Po-210 (**Figure 4**), i.e. the decay rate of a parent radionuclide equals that of its short-lived daughter/s, maintaining constant activity levels.

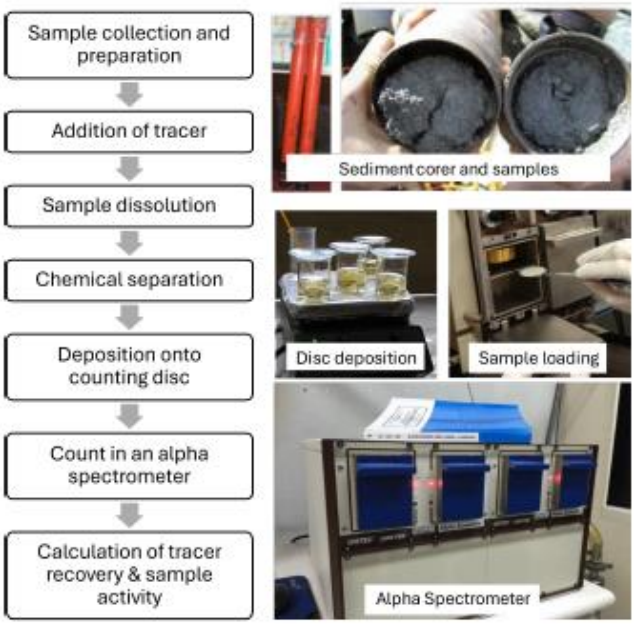


Figure 3. *Steps involved in radiochemical analysis by alpha spectrometry*

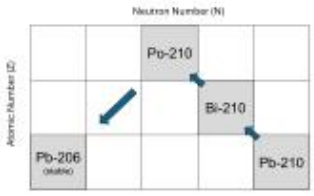


Figure 4. *Radioactive decay of Pb-210*

Sampling was conducted on April 29, 2009 (04/29/2009), where sediment core samples with approximately 150.0 cm length and 7.5 cm diameter were obtained from specific areas of Sorsogon Bay. The core was sectioned into 1 cm segments, which are then dried and homogenized. One (1) g of the sediment sample was digested by adding concentrated nitric acid and hydrofluoric acid. The extracted isotopes were plated onto silver disc from 0.5 $\mathrm{moles\ \ per\ \ liter}$ hydrochloric acid solution with ascorbic acid and hydroxylamine hydrochloride to reduce the effect of competing ions that are present in the sample. The isotope activities were then measured using an alpha spectrometry system with surface-barrier Si detector for a minimum of 24 h. The background count rates were determined. The data are collected and tabulated in the Excel file provided. The activity can be calculated by using the equation:

$$A_t = A_0 exp(-\lambda t) \quad (1)$$

where A_0 is the initial activity, A_t is the activity at time t (the time elapsed) and λ is the decay constant related to the half-life $\left(t_{1/2}\right)$, as given by:

$$\lambda = \ln(2)/t_{1/2} \quad (2)$$

Experiment Procedure

Part 1. Calculation of Tracer Activity and Relative Efficiency (2.1 pts)

Tracers are radionuclides that are added to the sample to quantify the relative efficiency of detection. They are assumed to behave in the same way as the radionuclide to be determined. The best tracers are relatively long-lived alpha-emitting isotopes that emit alpha radiation of different energy from the analyte nuclides.

During sample processing, a Po-208 tracer (half-life = 2.93 years) was added to calculate relative efficiency of the measurement. This is given by:

$$Relative\ Efficiency = A_{exp}/A_{calc} \quad (3)$$

where A_{exp} is the experimental activity calculated from the spectrometer counts, while A_{calc} is the calculated activity based on the known activity of the tracer. Starting with Core-1, 0.20 mL of Po-208 solution (activity concentration = 1.33 Bq/mL as of March 17, 2006 (03/17/2006)) was added to each segment to be extracted together with Po-210.

a. In **"Tab (1.1) Decay Constants"**, calculate the decay constant (λ) for Pb-210, Po-210 and Po-208 (Column D). These values will be used for the rest of activity/concentration calculations.

1.1	Write the **decay constant** (λ) calculated for Pb-210, Po-210, and Po-208.	**0.3 pt**

b. The data for Core-1 is shown in **"Tab (1.2) Tracer Activity"**. Calculate the tracer concentration in Bq/mL (Column C) and disintegration per minute (dpm)/mL (Column D) at the time of plating. Note: 1 year = 365.25 days

1.2	What is the average tracer concentration detected in the data set and its standard deviation?	**0.5 pt**

Shown in **Figure 5** are alpha spectra obtained from selected sections of the sediment core. The Po-208 is seen to peak at 5114.90 keV and Po-210 at 5304.38 keV. Each spectrum also indicates the total counts for both radionuclides after a specific counting time (t).

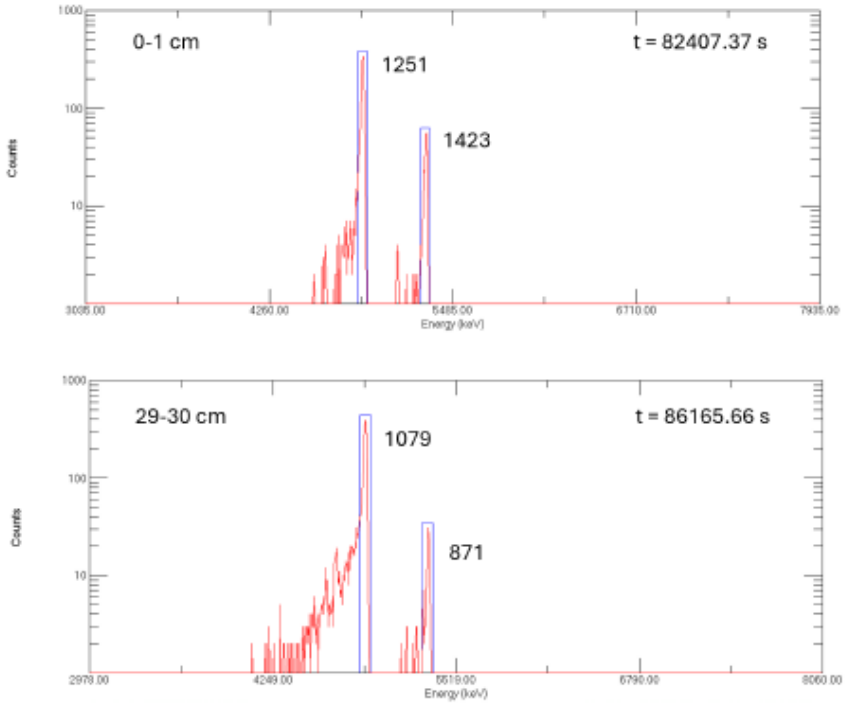


Figure 5. *Alpha spectra of two (2) segments from Core-1 showing the peak counts and total counting time (t)*

c. The total counts of Po-208, C_t, measured for each layer of Core-1 are given in **"Tab (1.3) Relative Efficiency"**. Determine the actual/experimental activity (Column H) of the tracer added using the given data and correct for background activity. Note: 1 count = 1 disintegration.

d. Recalculate the tracer concentration (in dpm, Column I) to account for the time elapsed between plating and end of sample counting. This allows for their comparison with the experimental data.

e. Determine the relative efficiency of the tracer added for each layer (Column J).

1.3	**(a)** Write the experimental tracer activity concentration values for the cores with spectra provided in **Figure 5**.	**0.8 pt**
	(b) What is the average relative efficiency of the entire dataset and its standard deviation?	**0.5 pt**

Part 2. Lead-210 Determination (2.3 pts)

As mentioned previously, the total Pb-210 can be determined from the activity of Po-210, assuming it is in secular equilibrium with its daughter nuclide Po-210.

a. The total counts of Po-210, C_X measured for each layer of Core-1 is given in **"Tab (2.1) Total Pb-210"**.
b. Determine the total Po-210 activity (in dpm, Column I) and activity concentration (in Bq/kg, Column J) at the time of plating.
c. From values obtained above, determine the total Pb-210 (in dpm and Bq/kg, Columns K and L) for each segment at the time of sampling.
d. Calculate the associated relative error (Column M) of the analysis in terms of ±Bq/kg (Column N) using the equation below:

$$Relative\ Error = 2\ sqrt\left(\frac{1}{C_x} + \frac{1}{C_t}\right) \qquad (4)$$

2.1	Complete the table with the calculated values for the core layers with spectra provided in **Figure 5**.	**1.2 pts**

The amount of *'supported' Pb-210* can be determined by the almost constant value of Pb-210, which is typically found in the lower region of the sediment core as shown in **Figure 1.**

e. Plot total Pb-210 versus depth along the sediment core in **"Tab (2.2) Pb-210 Plot"** to show the concentration of Pb-210 across the core. Use the average of the depth range for the depth data.
f. Use the relative error data to indicate error bars.
g. From the graph, calculate *'supported' Pb-210* using the average of the lower portion of the graph where the activity is no longer decreasing with depth. The range can be determined by using the set of consecutive values with the slope closest to 0, that is, no significant increase or decrease is observed.

2.2	Write the calculated 'supported' Pb-210 and corresponding standard deviation.	**1.1 pts**

Part 3. Lead-210 Geochronology (4.6 pts)

Pb-210 chronologies and sedimentation rate calculations can be estimated using different models, one of which is the *Constant Initial Concentration (CIC)* model. This model assumes a constant flux of the 'excess' Pb-210 such that at each stage in accumulation, the initial concentration of Pb-210 in the sediment is constant. The activity concentration, A, values of Pb-210 in undisturbed cores must decline monotonically with depth based on the radioactive decay as described below:

$$-\frac{dA}{dt} = \lambda A \tag{4}$$

Integrating this with the limits set by two points, the relationship between age and activity can be derived:

$$-\ln\left(\frac{A_2}{A_1}\right) = \lambda(t_2 - t_1) \tag{5}$$

For a constant sedimentation rate S, we can replace the depth axis x, with a time axis t, $S = x/t$. Substituting sedimentation rate in Equation 5 gives:

$$-\ln\left(\frac{A_2}{A_1}\right) = \frac{\lambda(x_2 - x_1)}{S} \tag{6}$$

Given that *sedimentation rate*, S is measured by depth $(x_2 - x_1)$ over a period of time $(t_2 - t_1)$, this can be further substituted to get an expression for sedimentation rate:

$$S = -\lambda\left[\left(x_2 - x_1\right) / \ln\left(\frac{A_2}{A_1}\right)\right] \quad (6)$$

Experimentally, the sedimentation rate can be determined by the least squares fit.

3.1	**(a)** On the grid provided, plot the logarithm of 'excess' Pb-210 values (in Bq/kg) versus sediment core depth.	**1.4 pts**
	(b) Use the least squares method to fit a straight line to the semi-logarithm plot for the top layer, from **0~20 cm**. Draw the corresponding best fit line in 3.1a. Complete the table with the calculated values from the least squares method and write down the linear equation, slope (m) and intercept (b) generated.	**0.7 pt**
	(c) Based on the slope obtained from 3.1b, what process likely explains the values obtained for this top layer?	**0.2 pt**
	(d) Use the least squares method to fit a straight line to the semi-logarithm plot for the bottom layer, from **20~60 cm**. Draw the corresponding best fit line in 3.1a. Complete the table with the calculated values from the least squares method and write down the linear equation, slope (m) and intercept (b) generated.	**0.7 pt**
	(e) Use the values obtained from 3.1d to determine the sedimentation rate of this layer.	**0.8 pt**

3.2	Determine the age (in years) of the sediment layer at **39.5 cm** at the time of sampling using the assumptions of the CIC model applied on the relevant region on the plot.	**0.8 pt**

Part 4. Cesium-137 Validation (1.0 pt)

Cesium-137 (half-life = 30.05 years) is produced by nuclear fission and has been released into the environment as a result of nuclear weapon testing during 1950–1970 (with a maximum atmospheric input in 1963) and the Chernobyl accident in 1986. It is considered an absolute dating technique that postdates these fallout events appearing as twin spikes on a profile graph. Dating using Cs-137 is usually performed to support and/or validate Pb-210 dating profiles.

a. In **"Tab 4.1 Cs-137 Plot"**, plot the measured Cs-137 activity for Core-1 versus the sediment core depth
b. Identify which depth can be dated as 1986 and 1963.

4.1	Calculate the sedimentation rate based on the two points with known dates.	**0.5 pt**

The same methods above were performed on five (5) more core samples within the same area, where we can assume comparable sedimentation conditions. The sedimentation rates are listed in **Table 1.**

Table 1. *Sedimentation rate obtained from Pb-210 and Cs-137 method*

Sample	Pb-210	Cs-137
Core-1	Value from 3.1e	Value from 4.1
Core-2	0.6213	0.5205
Core-3	0.5206	0.4309
Core-4	0.4911	0.4175
Core-5	0.5706	0.5304
Core-6	0.5516	0.5087

To check whether the Cs-137 method agrees with the data from Pb-210 method, a statistical test can be used to compare their means. The paired t-test can be used to determine whether the difference between the sample mean of two data sets performed on the same samples is significant or not. This is given by the equation:

$$t_{calc} = \left(\overline{x}_1 - \overline{x}_2\right) / \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \qquad (8)$$

where $\overline{x}$ is the mean or average, s is the standard deviation and n is the number of samples of of the data sets 1 and 2. When $t_{calc} < t_{tab}$ at a given level of confidence, it means there is no significant difference between the two methods.

c. Calculate the mean $(\overline{x})$ and standard deviation (s) for the data obtained from each method (**Table 1**). Treat Pb-210 as data set 1 and Cs-137 as data set 2.
d. Determine the t_{calc} between the two methods.
e. Compare with the t_{tab} found in **"Tab 4.2 t-tab values"** at 95% confidence level, for the appropriate degrees of freedom (total number of samples minus one).

4.2	**(a)** Write down the t_{calc} obtained.	**0.3 pt**
	(b) Based on the calculated t_{calc}, does the Cs-137 data validate the sedimentation rate obtained from Pb-210?	**0.2 pt**

Experiment 1 Scoring Guide

1st INSO International Nuclear Science Olympiad PHILIPPINES 2024

Scoring Guide for Q1 EXPERIMENTAL

Q1 ANALYSIS OF SEDIMENTATION PATTERNS USING LEAD-210 (10 pts)

Part 1. Calculation of Tracer Activity and Relative Efficiency (2.1 pts)

1.1

Isotope	λ		
Pb-210	0.0311	y^{-1}	0.1 pt
Po-210	0.0050	d^{-1}	0.1 pt
Po-208	0.2366	y^{-1}	0.1 pt

Accept variation in the decimal places in red

1.2

Average: 0.5679 Bq/ml or 34.0718 dpm/ml — 0.3 pt
Accept 0.56~0.57
Standard Deviation: 0.0028 Bq/ml or 0.1664 dpm/ml — 0.2 pt
Accept 0.002 – 0.003

1.3 (a)

Depth range, cm	Po-208 Counts	Experimental Tracer Activity, dpm
0 – 1	1251	0.9068
29-30	1079	0.7473

0.8 pt (0.2 pt each)

Accept variation in the decimal places in red

1.3 (b)

Average relative efficiency: 0.1262 — 0.3 pt
Standard Deviation: 0.0323 — 0.2 pt
Accept variation in the decimal places in red

Part 2. Lead-210 Determination (2.3 pts)

2.1

Depth range, cm	Total Po-210 Activity, Bq/kg	Total Pb-210 Activity, Bq/kg	Error, ± Bq/kg
0 – 1	122.6525	124.4292	9.6450 ~10
29 – 30	86.4196	87.6715	7.9870 ~8

Accept variation in the decimal places in red

1.2 pts (0.2 pt each)

2.2

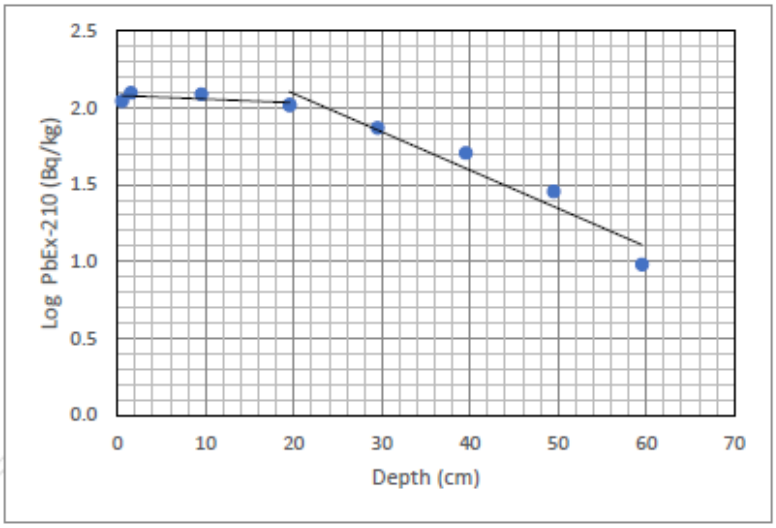
Supported Pb-210: 13.6352 Bq/kg — 0.8 pt

Standard Deviation: 4.4691 Bq/kg — 0.3 pt

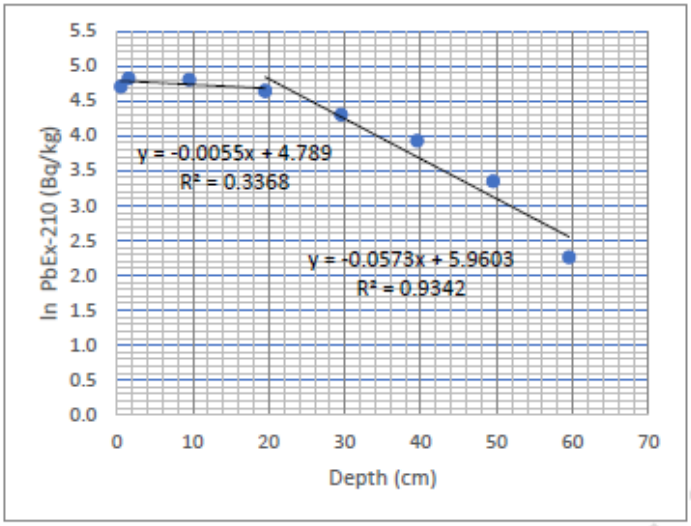
Accept variation in the decimal places in red

Part 3. Lead-210 Geochronology (4.6 pts)

3.1 (a)



1.4 pts (Axis label = 0.1 pt each
Data points = 0.05 pt each
Fit lines = 0.4 pt each



3.1 (b) — 0.7 pt

Depth (x)	LogPbEx-210 (y)	xy	x^2
0.5	2.0445	1.0223	0.2500
1.5	2.0957	3.1436	2.2500
9.5	2.0866	19.8229	90.2500
19.5	2.0179	39.3482	380.2500
$\Sigma x = 31.0$	$\Sigma y = 8.245$	$\Sigma xy = 63.337$	$\Sigma x^2 = 473.0000$

m = -0.0024
b = 2.0798
Linear equation: y = -0.0024x + 2.0798
Accept variation in the decimal places in red

(0.1 pt each)

Depth (x)	LnPbEx-210 (y)	xy	x^2
0.5	4.7775	2.3888	0.2500
1.5	4.7168	7.0752	2.2500
9.5	4.7024	44.6728	90.2500
19.5	4.4904	87.5628	380.2500
$\Sigma x = 31.0$	$\Sigma y = 18.6871$	$\Sigma xy = 141.6996$	$\Sigma x^2 = 473.0000$

m = -0.00134
b = 4.7758
Linear equation: y = -0.00134x + 4.7758
Accept variation in the decimal places in red

3.1 (c)	Bioturbation or mixing	0.2 pt

3.1 (d) — 0.7 pt

x	Y (log)	xy	x^2
19.5	2.0179	39.3482	380.2500
29.5	1.8694	55.1486	870.2500
39.5	1.7065	67.4068	1560.2500
49.5	1.4557	72.0579	2450.2500
59.5	0.9812	58.3809	3540.2500
Σx = 197.5	Σy = 8.0307	Σxy = 292.3424	Σx^2 = 8801.2500

m = -0.0249
b = 2.5885
Linear equation: y = -0.0249x + 2.5885
Accept variation in the decimal places in red

(0.1 pt each)

x	Y (ln)	xy	x^2
19.5	4.4904	87.5628	380.2500
29.5	4.3046	126.9844	870.2500
39.5	3.9294	155.2100	1560.2500
49.5	3.3519	165.9194	2450.2500
59.5	2.2593	134.4270	3540.2500
Σx = 197.5	Σy = 18.3355	Σxy = 141.6996	Σx^2 = 8801.2500

m = -0.0541
b = 5.8060
Linear equation: y = -0.0249x + 2.5885
Accept variation in the decimal places in red
*also accept 4-point data range below

30-60 cm (log)

Σx	Σy	Σxy	Σx^2
178.0	6.0129	252.9942	8421.0000
m	b		
-0.0292	2.8006		

20-50 cm (log)

Σx	Σy	Σxy	Σx^2
138.0	7.0495	233.9615	5261.0000
m	b		
-0.0185	2.4004		

20-60 cm (ln)

Σx	Σy	Σxy	Σx^2
197.5	18.3355	670.1035	8801.2500
m	b		
-0.0541	5.8060		

30-60 cm (ln)

Σx	Σy	Σxy	Σx²
178.0	13.8451	582.5407	8421.0000
m	b		
-0.0671	6.4487		

3.1 (e)	Sedimentation Rate: 0.5427 cm/y *Acceptable answers:* 0.4706 cm/y $<$ S $<$ 0.6409 cm/y	0.8 pt

3.2	23 years	0.8 pt

Part 4. Cesium-137 Validation (1.0 pt)

4.1	0.4348 cm/y Acceptable answers: 0.43-0.44	0.5 pt

4.2 (a)	2.8812 Accept variation in the decimal places in red	0.3 pt

4.2 (b)	No	0.2 pt

Points: 20

Time: 3.5 Hours

Q2 SHIELDING ACTIVATED MATERIAL (10 pts)

In this experiment, you will use computer applications that provide nuclear and radiation data on neutron activation analysis, and radiation shielding and protection scenarios. You will retrieve data from **JANIS**, a nuclear database software, to calculate the activity of a sample that is activated in a research reactor. You will also use the **EpiXS**, a photoatomic database software, to evaluate three different radiation shielding materials. Finally, you will optimize the thickness of a Lead (Pb) shielding while considering the effects of radiation buildup in a shielding material.

A. Experiment Background

A.1 Neutron Activation

Neutron irradiation plays a crucial role in advancing technology, medicine, science, and industry by enabling accurate isotopic analysis, material testing, high-grade silicone doping, and other innovative applications. To date, the strongest continuous source of neutrons for neutron irradiation applications are nuclear research reactors. During neutron irradiation, samples and instruments are placed into a research reactor core. Neutron activation may occur as the nuclides in the irradiated material absorb neutrons and become activated, producing radionuclide daughters.

The cross section (σ) of a nuclide measures the probability of its interaction with neutrons and has a unit of barns (b), with $1\ \mathrm{b} = 10^{-24}\ \mathrm{cm}^2$. The total cross section (σ_T) is the total probability of scattering and absorption interaction between a neutron and a target nuclide. Scattering interactions may be elastic or inelastic, while neutron absorption may result in radiative capture, neutron multiplication, charge multiplication, or nuclear fission among others. Neutron cross section is energy dependent and is unique for different parent nuclides. But a general trend is observed where interaction probability tends to be higher for low energy or thermal neutrons $(< 0.5\ \mathrm{eV})$, with resonance absorptions observed in the epithermal energy range $(0.5\ \mathrm{eV} - 10\ \mathrm{keV})$, while inelastic scattering tends to have higher probabilities for fast neutrons $(> 10\ \mathrm{keV})$. For this experiment, we will deal with radiative capture reaction where a target nucleus absorbs a neutron, becomes a radioactive nuclide, and decays at a specific half-life while emitting a gamma ray.

The saturated activity A_∞ of a neutron-activated material, defined as the maximum activity that is attained after very long irradiation times (i.e. ~5 times the half-life of the daughter nuclide), can be determined from the neutron flux (ϕ_i) in the irradiation location:

$$A_\infty = N(\phi_{th}\,\sigma_{th} + \phi_{epi}\sigma_{epi} + \phi_{fa}\sigma_{fa}) \qquad (1)$$

where σ_i is the target nuclide cross-section, 'th', 'epi', and 'fa' refers to thermal, epithermal, and fast neutrons, respectively, while N is the total number of target atoms of the target nuclide calculated as follows:

$$N_i = \frac{m f_i N_A}{M} \quad (2)$$

In equation (2), m is the mass of the element, f_i is the natural abundance and M is the atomic mass of the target nuclide i, and $N_A = 6.02214 \times 10^{23}\ \mathrm{mol}^{-1}$ is the Avogadro's number. For samples retrieved before the A_∞ is reached, a decay correction is applied to equation (1) to obtain the prompt activity $A(t)$ of the radionuclide after the irradiation time t:

$$A(t) = A_\infty \left(1 - e^{-\lambda t}\right) \quad (3)$$

where λ is the decay constant of the product nuclide.

A.2 Radiation Shielding

The gamma radiation emitted from activated materials is potentially hazardous to radiation workers handling them. Radiation shielding is typically incorporated to reduce unnecessary exposure to radiation and protect people and the environment from its harmful effects. Radiation shielding materials are used to absorb or scatter radiation, thereby reducing the amount that can reach and harm living tissue. The effectiveness of the shielding depends on the type and energy of the radiation and the properties of the shielding material.

The Beer-Lambert law describes how a beam of photon radiation, like gamma or x-rays, with initial intensity I_0weakens as it travels through a shielding material. When a buildup factor B is incorporated, it accounts for the additional intensity from scattered radiation, providing a more comprehensive description of the radiation's behavior in real-world scenarios:

$$I = B I_0 e^{-\mu x} \quad (4)$$

In equation (4), I is the attenuated intensity of radiation after incorporating a shielding material with a linear attenuation coefficient μ and thickness x. The μ is the probability of photon interaction per unit thickness of a material and it varies with the type of material and photon energy. Equation (4) also accounts for scattered radiation that may potentially increase the I by including the buildup factor (B). This factor represents the additional intensity beyond what would be expected from direct attenuation alone, which is a case when $B = 1$. The buildup factor depends on the shielding material, radiation energy, and the mean free path (mfp) of the photon in the material, which is inversely related to the linear attenuation coefficient ($\mu = 1/mfp$).

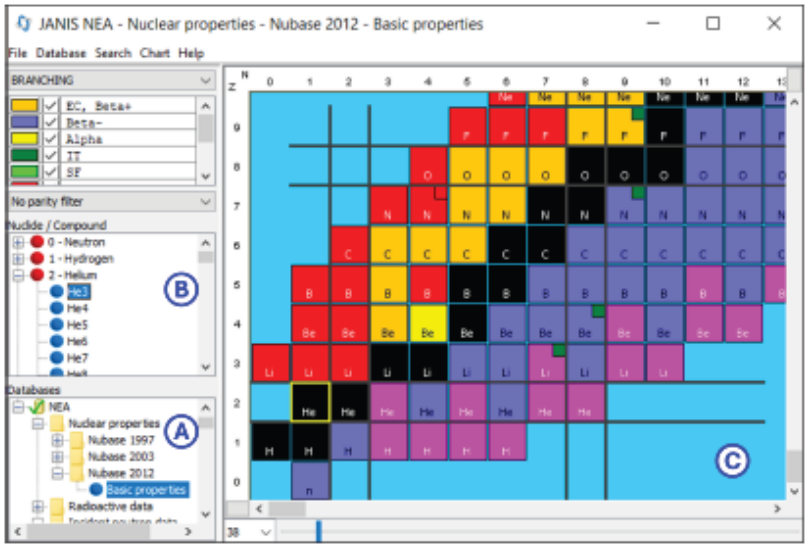
B. Software Description and Guide

B.1 JANIS (Java-based nuclear information software) is a software tool designed for managing and visualizing nuclear data. It provides an interface for accessing and analyzing various types of nuclear information, such as cross-sections, decay data, and fission product yields. JANIS is commonly used in nuclear physics and engineering for research, educational purposes, and data analysis, supporting various formats and sources of nuclear data.

Initiate the software by double-clicking or opening the **"Janis.jar – Shortcut"** in the desktop.

***Figure 1**. JANIS Icon*

Once the program is opened, the following window/interface should appear:



A **Database List.** Select the database you need here. Data include incident neutron cross-section data from different libraries (e.g. ENDF/B-VII.1, JEFF-3.0) and nuclear properties (e.g. Nubase 2012).

B **Nuclides/Compound selector.** The nuclide/ compound of interest may be selected here by "double-clicking"

C **Nuclide chart.** The nuclide/compound of interest may also be selected here by "double-clicking"

***Figure 2**. JANIS Interface*

When using the database for nuclear properties, after selecting a certain nuclide/compound, a new app window will be opened with an example shown in **Figure 3**.

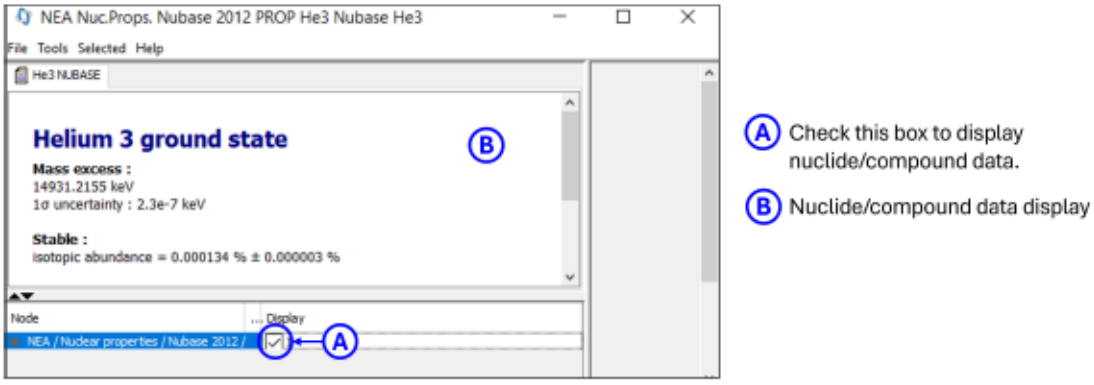


***Figure 3**. Sample software window for nuclear properties.*

When using the database for incident neutron data, after selecting a certain nuclide/compound, a new app window will be opened with an example shown in **Figure 4**.

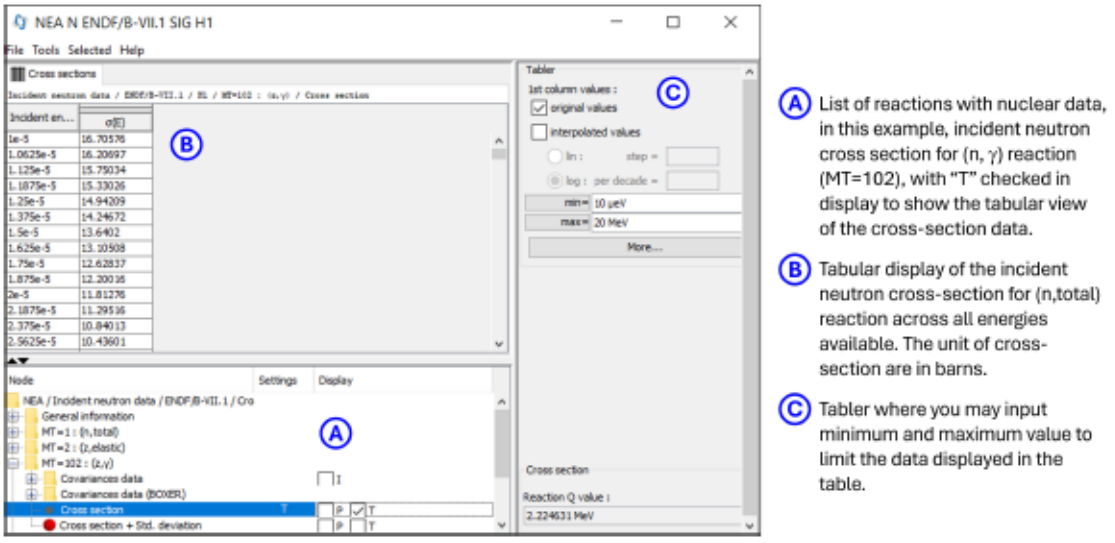


***Figure 4**. Sample window for incident neutron data.*

B.2 EpiXS is a Windows-based application software developed by the Philippine Nuclear Research Institute. It provides a manageable database that can be applied for photon attenuation, dosimetry, and shielding. The software incorporates photo atomic data of EPICS2017 from ENDF/B-VIII.0 and EPDL97 from ENDF/B-VI.8. It features data library interpolation in the energy range of 1 keV to 100 GeV and can calculate various parameters such as: partial or total cross sections (σ), mass attenuation coefficients (μ/ρ), linear attenuation coefficients (μ), mean free paths (mfp), half-value layers (hvl), effective atomic numbers (Z_{eff}), and electron densities (N_{eff}).

Initiate the EpiXS software by tapping on the icon displayed below:

***Figure 5**. EpiXS Icon*

The main menu interface will be displayed and by selecting "**Agree and enter**", access to the main simulator is granted. Upon choosing "**Enter composition**" in the subsequent interface, a prompt will appear for the material type: **element, compound, or mixture**. This can be selected as illustrated below.

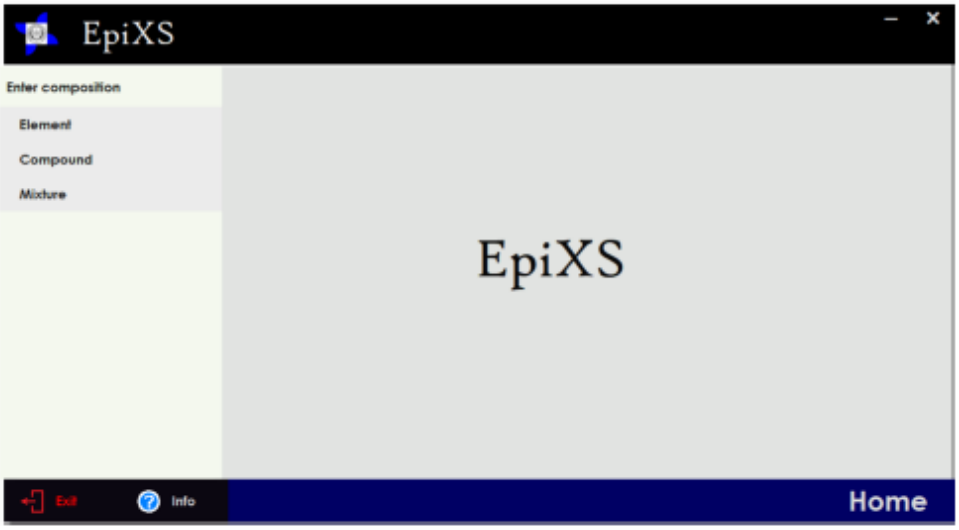


***Figure 6**. EpiXS Interface*

- **Elements** – Elements can be chosen either by their atomic number or their chemical symbol.
- **Compound** – For compounds, chemical formulas should be provided in standard chemical notation. Subscripts are required to be written in line. As an example, the formula for dihydrogen monoxide, commonly known as water, should be entered as H2O. Please note that the inclusion of parentheses, spaces, and dots is not supported by the software.
- **Mixture** – Mixtures can be composed of either "elemental" or "compound" components, or both. Users are required to provide the chemical symbol or formula (as outlined above), along with the weight fraction for each component in the format: <*compound*><*space*><*weight*>.

Once the material type is selected and all required information is entered in the appropriate format, the "**Proceed**" button can be clicked. It is important to note that providing the material's density is optional. However, without this parameter, calculations for the μ, mfp and hvl cannot be performed.

In the next interface, the computed parameters can be viewed either as a graph or as data, based on preference.

Experiment Scenario

Consider a 350 g stainless-steel sample that will be irradiated for 5 hrs in a material testing research reactor with a fully thermalized (average energy of 0.025 eV) neutron field with a flux of $\phi = 2.460 \times 10^{14}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$. The sample will be collected for characterization, and you are tasked to conduct a preliminary safety analysis in handling the irradiated sample. The elemental composition of the stainless-steel sample is provided in **Table 1** as well as the specific nuclides in the sample that will be most likely activated.

***Table 1**. Elemental compositions of stainless-steel weight*

Composition	Weight Percent (%)	Target Nuclei	Molar Mass (g/mol)	Main Activation Product
Ti	0. 03			
Ni	8.12			
Cr	8.0	Cr-50	49.946	Cr-51
Mn	14.0	Mn-55	54.938	Mn-56
Fe	68.85	Fe-58	57.933	Fe-59
Co	1.0	Co-59	58.933	Co-60

Experiment Procedure

Part 1. Data Collection (1.5 pts)

Use the following databases from the JANIS Software: (a) Nubase 2012 to gather atomic and nuclear data, and (b) ENDF/B VII.1 library for the cross-section data of the nuclides of interest.

1.1	Complete the table by indicating the radiative capture (MT = 102) cross-section (σ) in barns, and natural abundance (f_i) in %, of the nuclides of interest (Cr-50, Mn-55, Fe-58, Co-59), at the specific incident energy. Linear interpolatation of data may be needed for the required incident energy. For the σ, give the answer in 3 decimal places.	**1.0 pt**

1.2	Retrieve the half-life of each main activation product (Cr-51, Mn-56, Fe-59, Co-60) in seconds. Write the answers in proper scientific notation with 4 significant figures.	**0.5 pt**

Part 2. Neutron Activation (2.25 pts)

In **Table 2**, the activation products and their corresponding gamma radiation are given.

***Table 2**. Main activation products and radiation emitted*

Main Activation Product	Gamma Radiation Emitted (MeV)	Emission Probability (%)
Mn-56	0.847	98.85
	1.810	26.9
Fe-59	1.099	56.5
	1.292	43.2
Co-60	1.252*	199.83*

**Co-60 emits 2 gamma photons 1.33 MeV and 1.17 MeV at 99.98% and 99.85% emission probabilities, respectively. For many applications, it is a common practice to treat these photons as a single photon with an energy of 1.25 MeV (average energy of the 2 photons) at combined emission probability of 199.83% to simplify solutions.*

2.1	Calculate the prompt activity (A) in Bq of the nuclides listed in **Table 2** after 5-hour irradiation of the sample. Write the answers in proper scientific notation with 4 significant figures.	**1.5 pts**

Using point source approximation, the photon flux (φ) at a certain distance r from a gamma source can be obtained from source activity (A) and the emission probability (p):

$$\varphi = \frac{pA}{4\pi r^2} \quad (5)$$

2.2	Using the point source approximation, what are the flux of the gamma radiation with energies 0.847 keV, 1.099 MeV, and 1.252 MeV, assuming that the point of detection is 0.5 meters away and no gamma radiation attenuation and scattering. Give the answers in scientific notation with 4 significant figures.	**0.75 pt**

Part 3. Radiation Shielding (3.0 pts)

Once the stainless-steel sample is retrieved, it must be stored in a container that effectively shields against the gamma radiation to prevent unnecessary exposure. There are three available containers made from different proportions of Iron (Fe), Chromium (Cr), and Nickel (Ni), as detailed in **Table 3**. The sample, when placed inside, is positioned 0.5 meters from the container wall. Neglect gamma radiation attenuation in the air. You are tasked to evaluate these three containers and determine which one offers the best shielding function.

***Table 3**. Elemental compositions, thicknesses, and densities of the three containers.*

Containers	Elemental composition (wt%)			Thickness (cm)	Density (g/cm^3)
	Cr	Fe	Ni		
Alloy 1	18.0	72.0	10.0	2.1261	7.85
Alloy 2	21.0	46.5	32.5	2.0562	8.07
Alloy 3	25.0	55.0	20.0	2.1058	7.91

Use the EPICS2017 library of the EpiXS software. When extracting the parameters from the software, consider up to four decimal places for each value.

3.1	Asuming that there is no radiation buidup considered and all three containers have identical thickness. Determine the: **(a)** mass attenuation coefficient (probability of photon interaction per unit density of a material), **(b)** linear attenuation coefficient, and **(c)** mean free path of the container that provides the best shielding against a gamma energy of 1.099 MeV? Write all answers in 5 decimal places.	**0.9 pt**

3.2	If there is no buildup radiation considered and the activated sample is stored in the container that provides the best protection against gamma energy of 1.252 MeV, calculate the transmitted gamma radiation flux after this gamma energy passes through the container. Write the answer scientific notation with 4 significant figures.	**1.0 pt**

3.3	Assuming that there is no radiation buildup considered, find the ratio of the total transmitted to incident radiation after the three gamma energies have passed through Alloy 3 container. Write the answer in 3 decimal places.	**1.1 pts**

Part 4. Shielding Thickness Optimization (3.25 pts)

Radiation dose limits are established to protect people and the environment from the harmful effects of ionizing radiation by ensuring exposure remains within safe levels. Radiation workers have higher dose limits compared to the public because they are trained and monitored to handle radiation safely. The maximum permissible dose for radiation workers is $50\ \mathrm{mSv}$ in a year and a five-year average dose rate of $20\ \mathrm{mSv}\,/\,\mathrm{yr}$.

Radiation dose can be calculated from flux using flux-to-dose conversion coefficients. International organizations like the ICRP publish flux-to-dose conversion coefficients of different types of radiation based on dosimetry measurements and calculations. The table below shows the flux-to-dose conversion coefficients for gamma radiation.

***Table 5**. Flux-to-dose conversion coefficients for different gamma energies*

Gamma Energy (MeV	Conversion Coeff. (μSv/h)/(n/cm^2-s)
0.662	1.085
0.800	1.310
1.000	1.625
1.117	1.800
1.330	2.100

***Table 6.** Elemental composition and densities of the lead block and air*

Material	Elemental compoistion (wt%)	Density (g/cm^3)
Lead	100% Pb	11.348
Air	78% N; 21% O; 0.9% Ar	0.001225

4.1	Suppose a radiation worker is expected to handle activated samples for a total of 800 hrs in a year. For the purpose of this problem, let's assume that the dose rate received by the worker during these 800 hrs is constant. Calculate the maximum dose rate, in μSv/hr, that the worker can be exposed to without exceeding the five-year averaged dose limit.	**0.25 pt**

The buildup factor can be calculated for certain energies and will differ depending on the material used. The buildup factor can be calculated in different methods. One can solve the buildup factor as a function of μx, $B(\mu x)$, by using the Taylor's form given by:

$$B(\mu x) = Ae^{-\alpha_1 \mu x} + (1 - A)e^{-\alpha_2 \mu x} \tag{7}$$

where A, α_1, and α_2 are functions of energy, and x is the thickness. This function can sufficiently provide an accurate value for practical shielding problems. Values for A, α_1, and α_2 are different for each material and energies. Given in the table below is the values of A, α_1, and α_2 for lead. The parameters assume that the source is from a point isotropic source.

Table 7. *Parameters for the Taylor form of the Exposure Buildup Factor for Lead*

Energy (MeV)	A	$-\alpha_1$	α_2
0.5	1.677	0.03084	0.30941
1.0	2.840	0.03503	0.13486
2.0	5.421	0.03482	0.04379
3.0	5.580	0.05422	0.00611
4.0	3.897	0.08458	-0.02383
6.0	0.926	0.17860	-0.04635
8.0	0.368	0.23691	-0.05864
10.0	0.311	0.24024	-0.02783

Note: Linear interpolation should be applied for the flux-to-dose conversion coefficients corresponding to the energy of interest.

4.2 **(a)** Superimpose the two plots: **2.0 pts**

- Plot 1: The maximum desired gamma dose calculated from Question 4.1 (a constant value) as a function of μx.
- Plot 2: Calculated gamma dose being produced by the gamma source that has the highest contribution from Question 2.2 after it passes through a Pb block as a function of μx. Consider the contribution of buildup radiation.

Use μx values ranging from 17 to 19. Increment for μx may be as low as possible.

(b) Find the minimum thickness, in cm, of Pb that will provide the maximum allowed dose below the dose limit specified in Question 4.1. Give your answer in 3 decimal places. **1.0 pt**

Experiment 2 Scoring Guide

Q2 SHIELDING ACTIVATED MATERIAL (10 pts)

Part 1. Data Collection (1.5 pts)

1.1 — 1.0 pt (0.125 each)

Isotope	Cross-Section (barns)	Natural Abundance, (%)
Cr-50	15.500 to 15.505	4.345
Mn-55	13.361 to 13.400	100
Fe-58	1.157 to 1.160	0.282
Co-59	37.410 to 37.413	100

1.2 — 0.5 pt (0.125 each)

Isotope	Half-life (sec)
Cr-51	2.393×10^{6}
Mn-56	9.284×10^{3} to 9.288×10^{3}
Fe-59	3.844×10^{6} to 3.845×10^{6}
Co-60	1.662×10^{8} to 1.663×10^{8}

Part 2. Neutron Activation (2.25 pts)

2.1 — 1.5 pts (0.5 each)

Activated Product	Prompt Activity after 5 hours (Bq)
Mn-56	$(1.304 \pm 0.005) \times 10^{15}$
Fe-59	$(6.516 \pm 0.015) \times 10^{9}$
Co-60	$(2.469 \pm 0.001) \times 10^{10}$

2.2	Gamma Energy	Flux (#/cm^2-sec)	0.75 pts 0.25 /item
	0.847 MeV	$(4.103 \pm 0.015) \times 10^{10}$	
	1.099 MeV	$(1.172 \pm 0.003) \times 10^{5}$	
	1.252 MeV	$(1.570 \pm 0.001) \times 10^{6}$	

Part 3. Radiation Shielding (3.0 pts)

3.1	(a) $0.05743\ \text{cm}^2\text{g}^{-1}$	0.3 pt
	(b) $0.46343\ \text{cm}^{-1}$	0.3 pt
	(c) 2.15781 cm	0.3 pt
	*No or wrong units = wrong	

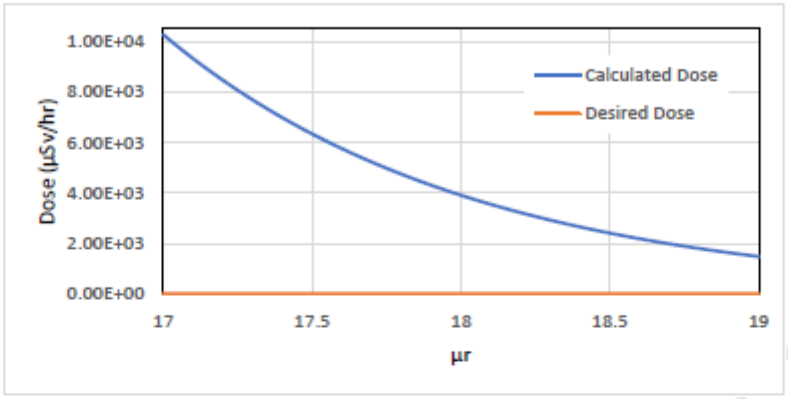
3.2	$I = 6.436 \times 10^{5}\ {}^{\text{photons}}/_{\text{cm}^2 \cdot \text{s}}$ Accept variation in the decimal places in red	1.0 pt

3.3	0.337 ± 0.002	1.1 pt

Part 4. Shielding Optimization (3.25 pts)

4.1	$25 \frac{\mu Sv}{hr}$	0.25 pt

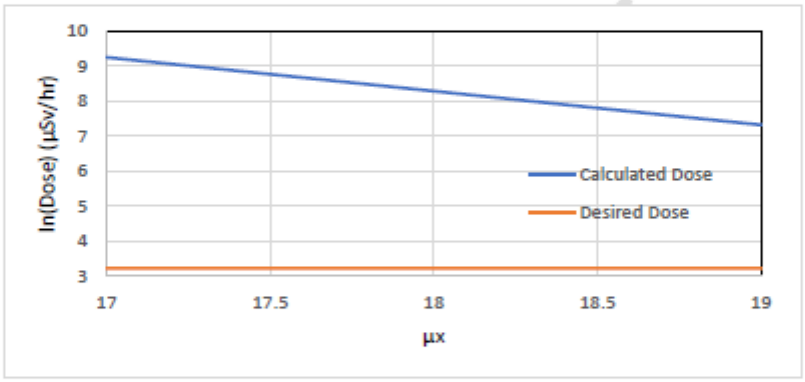
4.2 a) 2.0 pts



1.0 pt for straight line at *Dose* = 25 μSv/hr;

1.0 pt at exp plot with max of *Dose* = *210258* μSv/hr and min of *Dose* = *1490* μSv/hr;

or



1.0 pt for straight line at *Dose* = 3.22 μSv/hr;

1.0 pt at exp plot with max of *Dose* = *9.24* μSv/hr and min of *Dose* = *7.31* μSv/hr;

b) 24.629 cm or within the values [24.629 to 24.735 cm] 1.0 pt

Theoretical Questions

General Instructions

Points: 50

Time: 5.0 Hours

General Instructions

Failure to comply with any of the following instructions may lead to disqualification.

- The exam is worth a total of 50 points and will last 5 hours.
- The start and end times of the examination will be announced, hourly announcements will be made, and a reminder will be provided when there are 15 minutes remaining.
- Do not open the exam envelopes until instructed to do so.
- The following items are provided in your exam kit:
  - 1 ballpen
  - 1 pencil
  - 1 eraser
  - 1 ruler
  - 1 scientific calculator

During the exam:

- Use the ballpen provided. If you use the pencil to draft your notes, figures, tables, and graphs, make sure to trace the outlines of the final version with the ballpen.
- Use the Answer Sheets for your final answers. Fill in appropriate sections with your answers. Draw graphs as required. Cross out any unneeded answers.
- Blank working sheets are provided. Additional sheets are available upon request. Raise the "KIT" flag to notify the proctors.
- Keep your answers concise and legible. Use equations, operators, symbols, and sketches to convey your thoughts effectively.
- Uncertainty quantification is not required unless specified otherwise.
- Write numerical answers with 4 significant figures unless specified otherwise or when limited by the number of significant figures given in the problem. Use scientific notation as necessary, particularly for small (<0.01) and large numbers (>999).
- Do not leave your booth without permission. If you need a washroom break or other assistance, raise the appropriate flag(s) marked "Toilet", "Kit", "?", or "Medic".

At the end of the exam:

- Stop writing immediately when the end of the exam is announced.
- Place all answer sheets in the red envelope and seal. Sign across the envelop flap. Your proctor will assist you in securing the sheets in the envelope and will collect them afterwards.
- Place all the sheets (question paper, scratch paper, etc.) in their original envelope and leave on the table.
- Leave the exam kit on your table. You may take the remaining items with you, for example, the bottle of drinking water, and snacks.
- Your Proctor will let you know when you can leave.

Useful Information

Below is a list of information that could be helpful for your solution, in addition to what is provided in the questions.

Physical Constants

Constant	Symbol	Value
Avogadro's constant	N_A	$6.02214 \times 10^{23}\ \text{mol}^{-1}$
Boltzmann constant	k_B	$8.61733 \times 10^{-5}\ \text{eV K}^{-1}$
Coulomb's constant	k_e	$8.98755 \times 10^{9}\ \text{N m}^2\ \text{C}^{-2}$
Electron mass	m_e	$9.10938 \times 10^{-31}\ \text{kg}$
Elementary charge	q	$1.60218 \times 10^{-19}\ \text{C}$
Proton mass	m_p	$1.67262 \times 10^{-27}\ \text{kg}$
Speed of light in vacuum	c	$299\ 792\ 458\ \text{m s}^{-1}$

Conversion Factors

Quantity	Conversion Factors
Energy	$1\ \text{kWh} = 2.24694 \times 10^{19}\ \text{MeV}$ $1\ \text{eV} = 1.60218 \times 10^{-19}\ \text{joules (J) or kg m}^2\ \text{s}^{-2}$
Mass	$1\ \text{u} = 1.66057 \times 10^{-27}\ \text{kg}$
Mass-energy	$1\ \text{u} = 931.5\ \text{MeV}\ c^{-2}$
Nuclear cross-section	$1\ \text{barn (b)} = 10^{-28}\ \text{m}^2$

Formulas

Quantity	Formula	Definition of quantities
Bragg-Kleeman range	$R = N_R \times E_K{}^{\beta_e}$	N_R =proportionality factor E_k =kinetic energy β_e =exponent factor of incident energy
Centripetal force	$F_C = \frac{mv^2}{r}$	m =mass v =velocity r =radius
Electric field between parallel plates	$E = \frac{V}{d}$	V =potential difference between the plates d =distance between charges
Electric potential energy	$U = \frac{k_e q_1 q_2}{d}$	k_e =Coulomb's constant q_i =electric charge of particle i d =distance between charges
Kinetic energy	$E_K = qU$ $E_K = \frac{1}{2}mv^2$	q =elementary charge U =electric potential energy
Lorentz factor	$\gamma = 1/\sqrt{1 - \nu^2/c^2}$	v =velocity of particle
Magnetic Lorentz force	$F_B = qvB$	q =charge of particle B =magnetic field strength
Oscillation frequency	$f = \frac{1}{T}$	T =period
Period	$T = \frac{2\pi r}{v}$	
Radius of a nucleus	$r = r_0 A^{1/3}$	$r_0 = 1.2 \times 10^{-15}\mathrm{m}$ (empirical constant) A =atomic mass number

Quantity	Formula	Definition of quantities
Relativistic kinetic energy	$E_K = (\gamma - 1)m_0c^2$	γ =Lorentz factor m_0 =rest mass
Relativistic mass	$m_r = \gamma m_0$	m_r =relativistic mass of particle
Relativistic total energy	$E^2 = \left(m_0c^2\right)^2 + \left(pc\right)^2$	p =momentum of particle
Total energy	$E \equiv E_K + E_0 = E_K + m_0c^2$	E_K =kinetic energy of particle E_0 =rest mass energy of paticle m_0 =rest mass of paticle
Velocity of incident particle	$\beta_v \equiv \frac{v}{c} = \frac{pc}{E}$	

Q1 APPLICATION OF FUSION REACTION (10 pts)

Fusion reaction, where two nuclei are combined to form a heavier nucleus, is essential in understanding not only stellar mechanics but also for achieving controlled nuclear fusion as a clean and sustainable energy source on Earth. Among the nuclear reactions studied for these purposes, the Deuterium-Deuterium (D-D) and Deuterium-Tritium (D-T) neutron-producing fusion reactions are particularly significant. Only these two reactions will be considered throughout this problem.

Part 1. Energy from Fusion Reaction (2.0 pts)

Consider the following nuclear particles:

Table 1. *Nuclear particles involved in fusion reaction*

Particles	Symbol	Mass (u)
Deuterium (D)	${}_1^2\mathrm{H}$	2.014102
Tritium (T)	${}_1^3\mathrm{H}$	3.016049
Helium-3	${}_2^3\mathrm{He}$	3.016029
Helium-4	${}_2^4\mathrm{He}$	4.002603
Neutron	${}_0^1\mathrm{n}$	1.008665

1.1 **(a)** Calculate the Q-values in MeV for the D-D and D-T nuclear fusion reactions. **0.2 pt**

(b) The Q-value or the energy released from the reaction is split into the kinetic energies (E) of the reacting products. Derive an expression for the kinetic energy of the neutron (E_n) in terms of the masses of the reaction products (m_n and m_x) under the general reaction (reactants$\rightarrow X + n$) and the corresponding Q-value. Assume that the initial momentum is zero. **0.6 pt**

(c) From the expression obtained from (b), calculate the kinetic energy of the neutrons produced from the D-D and D-T reactions. **0.2 pt**

Fusion reaction was discovered in 1921 by Arthur Eddington, 17 years before the discovery of fission. However, while fission-based nuclear power plants are now supplying ~10% of the world's electricity since becoming commercially feasible by 1957, fusion reactors have yet to demonstrate their feasibility. Nonetheless, research and development in fusion technology is continued to ensure alternative clean energy for the future. The following question will compare the energy that can be derived from fusion and fission nuclear reactions with the energy derived from coal, which currently supplies ~36% of the world's electricity.

1.2 In 2022, it was reported that the worldwide average energy consumption per person is $3,600\ \mathrm{kWh}$.

(a) Assume that the reaction energy is fully recoverable for a fusion plant with an efficiency of 35%, how many kg of D-T reactants will be needed to provide $3,600\ \mathrm{kWh}$? How about D-D reactants? **0.5 pt**

(b) The average energy released per fission reaction is $200\ \mathrm{MeV}$. Assuming that this energy is fully recoverable, and that the fission plant has an efficiency of 30%, how many kg of fission reactants are needed to supply the same amount of energy? Note that the mass of $^{235}\mathrm{U}$ is $235.044\ \mathrm{u}$. **0.25 pt**

(c) How many kg of coal is needed to provide the same amount of energy if combustion of bituminous coal releases $31\ \mathrm{kJ\ per\ g}$ of fuel? **0.15 pt**

(d) What is the ratio of your answer for the mass of coal and the mass of D-T reactants from your answer in 1.2 (a)? **0.1 pt**

Part 2. Tritium Production (2.0 pts)

D-T fusion reaction is considered more feasible for fusion energy research due to its higher interaction probability and higher energy released per reaction compared to D-D fusion. Moreover, tritium, though not naturally abundant, can be bred from lithium. Lithium is more readily available than deuterium, which is the sole source for the D-D reaction. Natural lithium is 4.85% ^{6}Li and 95.15% ^{7}Li. Although tritium $^{3}_{1}H$ can be produced from both isotopes, the following ^{7}Li reaction is a threshold reaction, which requires fast neutrons. (Note: Percentages are isotopic abundances.)

$$^{7}_{3}Li + ^{1}_{0}n \rightarrow ^{4}_{2}He + ^{3}_{1}H + ^{1}_{0}n \quad (1)$$

Meanwhile, ^{6}Li is a strong neutron absorber which can produce tritium through the following (n,α) reaction with interaction cross section of $\sigma = 940$ barn.

$$^{6}_{3}Li + ^{1}_{0}n \rightarrow ^{4}_{2}He + ^{3}_{1}H \quad (2)$$

The total amount of $^{3}_{1}H$ that can be produced from reaction (2) can be calculated from the reaction rate R:

$$R = \sigma N \phi V \quad (3)$$

where ϕ is the neutron flux and V is the volume of the sample. Meanwhile the nuclide density (N) is shown below, which is expressed in terms of density (ρ), Avogadro's number (N_A) and atomic mass($m_{^{6}Li} = 6.015122$ u):

$$N = \rho N_A / m \quad (4)$$

2.1 Calculate the minimum incident neutron energy, in MeV, that is required to produce $^{3}_{1}H$ from ^{7}Li. Note that $m_{Li7} = 7.016003$ u. Express the answer with 4 significant figures. **0.8 pt**

2.2 **(a)** Calculate the mass (in g) of $^{3}_{1}H$ nuclides produced from 1 kg of natural lithium that is irradiated for 24 h in a fission reactor with a neutron flux of 3.5×10^{14} n cm^{-2} s^{-1}. Assume that the irradiator source consists of purely thermal neutron. **1.0 pt**

(b) A D-T fusion reactor with $1,000$ MW output will require approximately 150 kg of tritium fuel per year. If we can only irradiate natural lithium with thermal neutron once in a reactor for 24 h, how much ^{6}Li should we place inside to produce 150 kg of tritium required for annual operation? **0.2 pt**

Part 3. Overcoming the Coulomb barrier (2.5 pts)

Electric potential energy (U) is the total work done to bring a system of charges into a certain configuration.

3.1	Calculate the energy required to bring a deuteron at contact to a target: **(a)** deuteron for D-D reaction, and **(b)** triton for D-T reaction.	**1.6 pts**

Due to the positive charge of the nucleus, fusion reaction between light nuclei occurs only when the reacting particles are sufficiently energetic to overcome the Coulomb repulsive force. This requires heating the reactants to very high temperatures to attain the plasma state. The energy distribution of fusion reactants in plasma state is given by the Maxwell-Boltzmann distribution law, which relates the plasma temperature (T) to the average energy $\left(\overline{E}\right)$ of the particles as follows:

$$\overline{E} = \frac{3k_BT}{2} \tag{5}$$

where k_B is the Boltzmann constant. We assume that the particles undergo elastic collision. The actual ignition temperature for D-D and D-T reactions is around $4.5 \times 10^8\,\mathrm{K}$ and $1.5 \times 10^8\,\mathrm{K}$ respectively. It differs from the theoretical ignition temperature due to quantum tunneling and the wide range of energy of the reacting nuclei, which is described by the Maxwell-Boltzmann distribution.

3.2	**(a)** Determine the corresponding temperatures for the energies calculated for D-D and D-T reactions from 3.1.	**0.6 pt**
	(b) Calculate the percentage difference between the actual and theoretical ignition temperatures for the D-D and D-T reactions. Use the following formula for % difference: $\%diff = \frac{\lvert X_1 - X_2 \rvert}{\left(\frac{X_1 + X_2}{2}\right)} \times 100\%.$	**0.3 pt**

Part 4. Portable Neutron Generators (3.5 pts)

Although power reactors based on fusion reaction are not yet commercially available due to the very high temperature requirements for the reacting particles, D-D and D-T fusion reactions have been used in portable neutron generators (NG) for research and industrial applications. An important parameter for NG applications is the total neutron yield S:

$$S = N_s \sigma \varphi \tag{6}$$

where N_S is the surface density or the number of target nuclei per unit area of the target, σ is the reaction cross section, and φ is the incident particle rate, which can be expressed empirically in terms of the deuteron beam current (I) in amperes and the incident particle charge (q):

$$\varphi = \frac{I}{q} \bullet a \tag{7}$$

In equation 7, a is an empirically determined unitless parameter that is dependent on the configuration of a particular neutron generator unit. For an incident deuteron with $100\ \mathrm{keV}$ energy, the cross section for the D-D reaction is 16.9 millibarns while the corresponding cross section for the D-T reaction is 293 times higher.

Consider a $1\ \mathrm{mm}$ thick titanium layer that was subjected to hydridation until the titanium to hydrogen ratio reached 1:1.64. The characteristics of the titanium hydrides are summarized in **Table 2**.

Table 2. *Characteristics of hydridated titanium targets*

TiH species	Density, ρ	Molecular Mass
$TiD_{1.64}$	$3.92\ \mathrm{g\ cm^{-3}}$	$51.170127\ \mathrm{u}$
$TiT_{1.64}$	$4.03\ \mathrm{g\ cm^{-3}}$	$52.813320\ \mathrm{u}$

4.1	Calculate the surface density N_s of the D and T atoms of **(a)** $TiD_{1.64}$ and **(b)** $TiT_{1.64}$ respectively. Note that $N_s = N \cdot l$ where N is the atom density and l is the sample thickness.	**1.4 pt**

4.2	A particular neutron generator unit with targets as described in this problem was determined to have the parameter value $a = 6.25 \times 10^{-4}$. Calculate the required beam current (in amperes) for this $100\ \mathrm{keV}$ neutron generator to produce a neutron yield of $S = 1 \times 10^{7}\ \mathrm{n\ s^{-1}}$ from D-D reactions and D-T reactions.	**1.4 pt**

One application of the portable neutron generator is neutron radiography (NR), which uses a neutron beam to visualize the internal structures of materials. This technique is highly effective for materials with low atomic numbers. The aperture in the neutron generator where the generated neutrons exit is commonly referred to as the neutron exit port. For a neutron generator with an isotropic neutron yield (S), the flux (ϕ) at a distance which is significantly larger than the neutron exit port (r), can be approximated by:

$$\phi = \frac{S}{4\pi r^2} \tag{8}$$

For this neutron flux to reach an object to be radiographed, the NR system uses a collimator to guide the neutrons towards the object. The collimator typically resembles a truncated cone or pyramid and may have a circular, square, or rectangular cross-section. The emerging flux from the collimator outlet aperture (ϕ) can be estimated using the relation:

$$\phi = \frac{\phi_0 A}{4\pi L^2} \tag{9}$$

where ϕ_0 is the flux from the collimator inlet aperture, A is the area of the collimator inlet aperture, and L is the distance from the collimator inlet to the imaging plane.

4.3	**(a)** Calculate the ϕ if the inlet aperture of the collimator is $3\ \mathrm{cm}$ away from the neutron generator, as described in 4.2. Assume that the neutron exit port is significantly smaller than $3\ \mathrm{cm}$.	**0.2 pt**
	(b) Consider a circular collimator that initially has a diameter of D_1 with the corresponding ϕ_1. Suppose the diameter of this collimator is adjusted to D_2 while keeping the ϕ_0 constant. Write the expression for the flux ϕ_2 in terms of ϕ_1.	**0.5 pt**

Question 1 Scoring Guide

Q1 FUSION REACTION (10 pts)

Part 1. Energy from fusion reaction (2.8 pts)

1.1 (a)	D-D: 3.270 MeV D-T: 17.59 MeV	0.1 pt 0.1 pt
1.1 (b)	$E_n = \left(\frac{m_x}{m_x + m_n}\right) Q$	0.6 pts
1.1 (c)	D-D: $E_n = 2.450$ MeV D-T: $E_n = 14.05$ MeV	0.1 pt 0.1 pt
1.2 (a)	D-D: 4.728×10^{-4} kg $(4.680 \times 10^{-4} - 4.775 \times 10^{-4}$ kg$)$ D-T: 1.098×10^{-4} kg $(1.087 \times 10^{-4} - 1.108 \times 10^{-4}$ kg$)$ P *Note: Answers within ±1% of these values are acceptable.*	0.25 pts 0.25 pts
1.2 (b)	5.285×10^{-4} kg $(5.232 \times 10^{-4} - 5.337 \times 10^{-4}$ kg$)$ *Note: Answers within ±1% of this value are acceptable.*	0.25 pts
1.2 (c)	4.181×10^{2} kg	0.15 pt

1.2 (d)	3.808×10^6 $(3.769 \times 10^6 - 3.846 \times 10^6)$ *Note: Answers within ±1% of this value are acceptable.*	0.1 pt

Part 2. Tritium production (3.2 pts)

2.1	2.468 MeV (2.443 – 2.493 MeV) *Note: Answers within ±1% of this value are acceptable.*	0.8 pts

2.2 (a)	0.597 g of ^3_1H (0.590 – 0.603 g) *Note: Answers within ±1% of this value are acceptable.*	1.0 pt

2.2 (b)	2.513×10^5 kg $(2.488 \times 10^5 - 2.538 \times 10^5$ kg) *Note: Answers within ±1% of this value are acceptable.*	0.2 pts

Part 3. Overcoming the Coulomb barrier (4.0 pts)

3.1 (a)	7.630×10^{-14} N · m $(7.553 \times 10^{-14} - 7.706 \times 10^{-14}$ N · m) *Note: Answers within ±1% of this value are acceptable. Alternate unit is joules or J.*	0.8 pts

3.1 (b)	7.115×10^{-14} N · m $(7.044 \times 10^{-14} - 7.186 \times 10^{-14}$ N · m) *Note: Answers within ±1% of this value are acceptable. Alternate unit is joules or J.*	0.8 pts

3.2 (a)	D-D: 1.842×10^9 K $(1.824 \times 10^9 - 1.860 \times 10^9$ K$)$ D-T : 1.718×10^9 K $(1.701 \times 10^9 - 1.735 \times 10^9$ K$)$ *Note: Answers within ±1% of these values are acceptable.*	0.3 pts 0.3 pts

3.2 (b)	D-D: 121.5% (120.3 – 122.7%) D-T : 167.9% (166.2 – 169.6%) *Note: Answers within ±1% of these values are acceptable*	0.15 pts 0.15 pts

Part 4. Portable neutron generators (3.5 pts)

4.1	(a) D: $N_{s,D} = 7.566 \times 10^{21}$ cm^{-2} $(7.490 \times 10^{21} - 7.642 \times 10^{21}$ cm$^{-2})$ (b) T: $N_{s,T} = 7.536 \times 10^{21}$ cm^{-2} $(7.461 \times 10^{21} - 7.612 \times 10^{21}$ cm$^{-2})$ *Note: Answers within ±1% of these values are acceptable*	0.7 pts 0.7 pts

4.2	D-D: 2.005×10^{-5}A $(1.985 \times 10^{-5} - 2.025 \times 10^{-5}$ A$)$ D-T : 6.869×10^{-8}A $(6.801 \times 10^{-8} - 6.938 \times 10^{-8}$ A$)$ *Note: Answers within ±1% of these values are acceptable.*	0.7 pts 0.7 pts

4.3 (a)	8.842×10^4 n/cm$^2 \cdot$ s $(8.754 \times 10^4 - 8.930 \times 10^4$ n/cm$^2 \cdot$ s$)$ *Note: Answer within ±1% of this value is acceptable.*	0.2 pts

4.3 (b)	$\phi_2 = \frac{{D_2}^2}{{D_1}^2}\phi_1 = \left(\frac{D_2}{D_1}\right)^2 \phi_1$	0.5 pts

Q2 STERILE INSECT TECHNIQUE (SIT) FOR MOSQUITOES (10 pts)

Scientists combating mosquito-borne diseases such as malaria, dengue, West Nile virus, chikungunya, yellow fever and Zika are developing the use of the Sterile Insect Technique (SIT). This technique involves the mass breeding, sterilization, and release of sterile male insects into the field to mate with their female counterparts. The resulting offspring from this mating are not viable, thus reducing the population of the next generation through continuous release. In SIT, insects are rendered sterile by exposure to ionizing radiation such as gamma rays. Cobalt-60 or ^{60}Co can be used for this irradiation, which sterilizes without inducing radioactivity. It is essential to select a dose that effectively sterilizes male mosquitoes without significantly impacting their lifespan and mating competitiveness.

Part 1. Absorbed Dose Measurement using Fricke Dosimeter (4.5 pts)

In SIT, the irradiation dose delivered via ^{60}Co gamma rays plays a crucial role. The validation and routine control of this sterilization process relies on dosimetry, which is the measurement of the radiation dose received by the insects. Among the different types of dosimeters, Fricke dosimeter, commonly known as ferrous sulfate dosimeter, is one of the most versatile chemical dosimeters available. It works by tracking how ionizing radiation oxidizes ferrous ions $\left(Fe^{2+}\right)$ to ferric ions $\left(Fe^{3+}\right)$. The concentration of these ferric ions, which increases with radiation exposure, can be measured spectrophotometrically at $303\ \mathrm{nm}$. The Fricke dosimeter will be used to determine the absorbed dose of the mosquito pupae in a Petri dish.

The SI unit of absorbed dose is gray (Gy), which is defined as "the absorption of 1 joule of energy by 1 kilogram of matter". Suppose a radioactive material has an activity A, and each disintegration releases a gamma ray photon with an energy E.

1.1 Write an equation for the power (total energy released per unit time) from gamma rays, in $\mathrm{J\,h^{-1}}$, if A is reported in MBq, and E is expressed in MeV $\left(1\ \mathrm{J} = 6.2415 \times 10^{18}\ \mathrm{eV}\right)$. **0.5 pt**

1.2 Write an equation for the power density per unit area, in units of $\mathrm{J\,h^{-1}\,m^{-2}}$, at a distance r from the radioactive material source. Assume an isotropic gamma emission. **1.0 pt**

As the gamma ray photons travel from the source to the Fricke dosimeter positioned at a distance r meters away, they can be absorbed by the air. The attenuation due to this absorption of gamma ray photons per unit distance travelled to the Fricke dosimeter is given by the equation:

$$I = I_o e^{-\mu_a r} \quad (1)$$

where I_o is the initial gamma ray photon intensity, I is the gamma ray photon intensity after travelling a distance r, and μ_a is the linear attenuation coefficient for air, in m^{-1}.

The dimensionless quantity $e^{-\mu_a r}$ is the attenuation factor f; it gives the fraction of the initial gamma ray photons that travelled the distance r without being absorbed. An alternative form is the mass attenuation coefficient, $\frac{\mu}{\rho}$, which is the linear attenuation coefficient μ normalized to the density ρ of the material. We can provide a more general expression for the attenuation factor f for gamma ray photons that travelled a distance x meters through arbitrary matter:

$$f = e^{\left(-\left(\frac{\mu}{\rho}\right)\rho x\right)} \quad (2)$$

Assuming all attenuated photons' energies are fully absorbed by the material, the dose rate (DR, with units of $\mathrm{Gy\ h^{-1}}$) at a distance r from the source is given by:

$$DR\ (\mathrm{Gy\ h^{-1}}) = \frac{1.442\times10^{-4}AE}{\pi r^2}\left(\frac{\mu}{\rho}\right) \quad (3)$$

where A is expressed in MBq and E is in MeV.

1.3	Calculate the dose rate (in $\mathrm{Gy\ h^{-1}}$) and dose (in Gy) to the Fricke dosimeter positioned $50\ \mathrm{cm}$ away from a ^{60}Co source with an activity of $8000\ \mathrm{Ci}$ after exposure for 10 minutes. ^{60}Co produces gamma rays with an average energy of $1.25\ \mathrm{MeV}$. The mass attenuation coefficient for a standard Fricke (ICRU 44) is $0.072\ \mathrm{cm^2\ g^{-1}}$.	**1.5 pt**

Ionizing radiation oxidizes Fe^{2+} to Fe^{3+}, and the radiation chemical yield $G(Fe^{3+})$ was found to be proportional to the absorbed dose. The increase in the concentration of Fe^{3+}, compared to the concentration in the unirradiated solutions measured on the same day, is calculated from the increase in the optical density (OD) at $303\ \text{nm}$ at $25\ ^{\circ}\text{C}$. The absorbed dose to the Fricke dosimeter which was exposed to ionizing radiation is given by:

$$D = \frac{\Delta OD}{\varepsilon G(Fe^{3+})\rho l} \quad (4)$$

where ΔOD is the increase in optical density, ε is the extinction coefficient of Fe^{3+} minus the extinction coefficient of Fe^{2+} at $303\ \text{nm}$, $G(Fe^{3+})$ is the radiation chemical yield of Fe^{3+}, ρ is the density of the Fricke solution, and l is the length of the light path of the cuvette.

1.4	Using the exposed Fricke dosimeter described in 1.3, determine the radiation chemical yield for Fe^{3+} in $\mu\text{mol J}^{-1}$. The optical density measured at $303\ \text{nm}$ at $25\ ^{\circ}C$ was 0.1840 after the completion of irradiation and 0.0030 before irradiation. $\varepsilon = 2174\ \text{L mol}^{-1}\,\text{cm}^{-1}$, $\rho = 1.024\ \text{g cm}^{-3}$, $l = 1\ \text{cm}$.	**0.5 pt**

1.5	In a hypothetical scenario where each ^{60}Co gamma ray photon deposits $300\ \text{keV}$ of energy into a $1\ \text{g}$ Fricke dosimeter per interaction, how many photon interactions would be needed for the dosimeter to receive absorbed doses of $10\ \text{Gy}$, $20\ \text{Gy}$, $30\ \text{Gy}$, $40\ \text{Gy}$, and $50\ \text{Gy}$? These are doses that have been evaluated for use in the SIT for mosquitoes.	**1.0 pt**

Part 2. Egg Hatch and Dose in SIT (3.5 pts)

In the SIT, the egg hatch percentage plays a crucial role. This percentage represents the proportion of eggs that successfully develop into larvae and hence directly impacts the mosquito population. By increasing the irradiation dose, a lower egg hatch percentage can be achieved. The continuous release of sterile males results in fewer mosquitoes in subsequent generations. **Figure 1** represents a relationship between egg hatch percentage and irradiation dose.

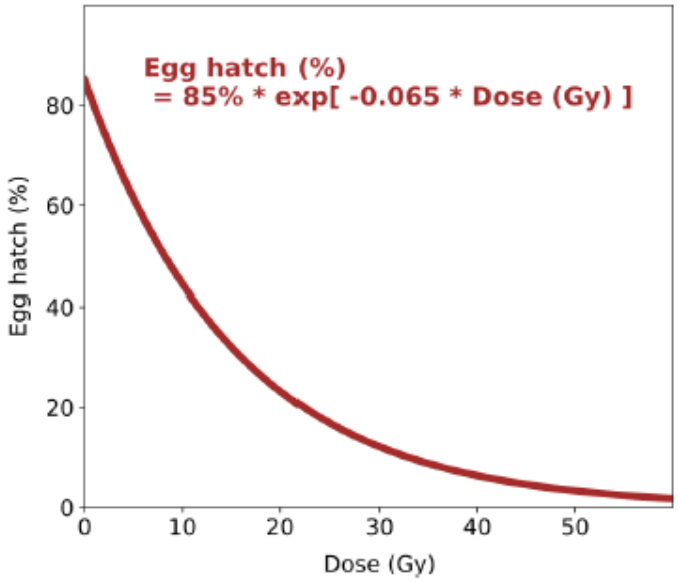


Figure 1. *Relationship between irradiation dose and egg hatch (%)*

A bare ^{60}Co source was used to irradiate a Petri dish of male mosquito pupae at a certain dose. However, laboratory tests revealed that the egg hatch of their offspring was only 40%. The calculated relationship between the egg hatch and the irradiation dose in Gy was:

$$Egg\ hatch\ (\%) = 85\% \times exp(-0.065\ Dose) \quad (5)$$

2.1	Based on equation 5 and the observed egg hatch, calculate the dose that was used to irradiate the mosquito pupae.	**0.5 pt**

2.2	If the irradiation dose that resulted in a 40% egg hatch was administered over a 10-minute period, what should have been the duration of irradiation to achieve a 1% egg hatch?	**1.0 pt**

The introduction of a new ^{60}Co source increased the dose rate, leading to a fixed one-minute irradiation period. This change resulted in a higher dose, which effectively reduced the egg hatch rate to 0.1%. However, this higher dose had a detrimental effect on the sterile male mosquitoes, compromising their mating competitiveness, a key factor for SIT's success. To maintain effective population control while ensuring the competitiveness of the sterile males, a lead shield is proposed with the aim of slightly increasing the egg hatch rate to 2%. This rate, while higher than 0.1%, still allows for effective population control, assuming the males remain competitive. The proposed lead shield could play a crucial role in fine-tuning the irradiation dose while preserving the quality and competitiveness of the sterile males.

The shielded dose can be calculated by the equation below:

$$D_i = D_0 exp(-(\mu/\rho)\rho x) \quad (6)$$

where D_i is the shielded dose in Gy, D_0 is the unshielded dose, (μ/ρ) is the mass attenuation coefficient of the material, ρ is the density of the material, and x is the thickness of the material.

2.3 Assuming no buildup factor effects from the proposed lead shield and that ^{60}Co gamma rays are monoenergetic at $1.25\ \mathrm{MeV}$, calculate the required lead thickness (in cm) to increase the egg hatch from 0.1% to 2%. Use a mass attenuation coefficient, $(\mu/\rho)_{Pb}$, of $0.058\ \mathrm{cm^2\ g^{-1}}$ for these rays in lead (with a density, ρ, of $11.3\ \mathrm{g\ cm^{-3}}$). **1.0 pt**

Given the availability and durability of $1\ \mathrm{mm}$ thick stainless-steel sheets in the facility, your team is considering stainless steel as an alternative to the proposed lead shield. The density of stainless steel is $8.03\ \mathrm{g\ cm^{-3}}$. **Table 1** provides the elemental mass attenuation coefficients, $(\mu/\rho)_i$, and weight fractions (w_i) of each element in stainless steel.

Table 1. *Elemental composition of stainless steel, showing the weight fraction (Wt%) and mass attenuation coefficient (MAC) for each element*

Elements	Wt%	MAC
C	0.0008	0.0568
Mn	0.02	0.0521
P	0.00045	0.0551
S	0.0003	0.0568
Si	0.01	0.0567
Cr	0.19	0.0528
Ni	0.095	0.0548
Fe	0.68345	0.0534

2.4 **(a)** Calculate the effective mass attenuation coefficient, $(\mu/\rho)_{ss}$, of stainless steel using the equation: **0.5 pt**

$$\left(\frac{\mu}{\rho}\right)_{ss} = \sum_i w_i \times \left(\frac{\mu}{\rho}\right)_i \qquad (7)$$

where w_i is the weight fraction and $(\mu/\rho)_i$ is the mass attenuation coefficient of the i-th element.

(b) Calculate how many $1\ \text{mm}$ thick stainless-steel sheets are needed to achieve the desired shielding effect that keeps the egg hatch rate close to 2%. Round up fractional results to use full $1\ \text{mm}$ sheets. **0.5 pt**

Part 3. Competitiveness and Dose in SIT (2.0 pts)

In the ongoing battle against mosquito-borne diseases, scientists have noticed an intriguing trend. As the irradiation dose increases, the mating competitiveness of the sterile male mosquitoes initially experiences a gradual decline. Competitiveness is defined as their ability to successfully compete with wild males for mating with wild females. However, beyond a certain dose threshold, this decline becomes more pronounced, leading to a sharp drop in competitiveness. This pattern is depicted in **Figure 2**, underscoring the critical role of precise dose management in optimizing the SIT for mosquito control. For the following problems, assume that the irradiation times are fixed.

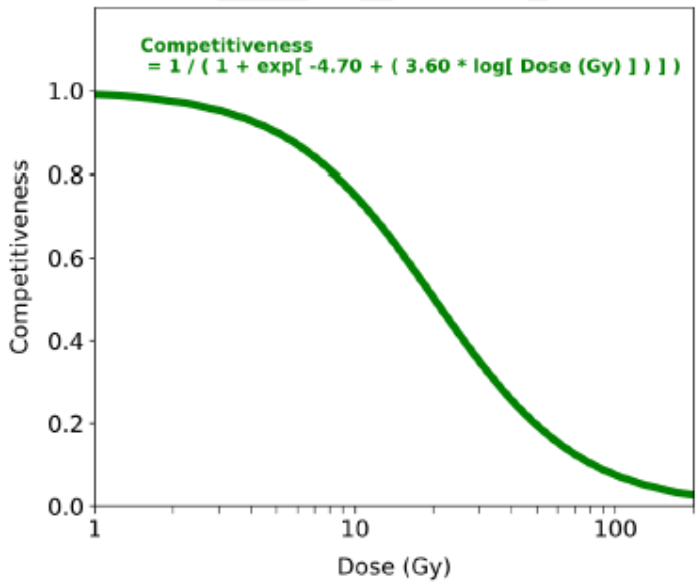


***Figure 2**: Relationship between irradiation dose and mating competitiveness*

3.1 Given the equation for male competitiveness (C) as a function of dose: **1.0 pt**

$$C = \frac{1}{1+exp(-4.7+[3.6\times\log(Dose)]} \qquad (8)$$

If an irradiation dose of 40 Gy is administered, calculate the resulting competitiveness.

3.2 Conversely, if specific competitiveness level of 0.5 is desired, calculate the irradiation dose required to achieve this level. **1.0 pt**

Question 2 Scoring Guide

1st INSO International Nuclear Science Olympiad PHILIPPINES 2024

Scoring Guide for Q2 THEORY

Q2 STERILE INSECT TECHNIQUE FOR MOSQUITOES (10 pts)

Part 1. Absorbed Dose Measurement Using Fricke Dosimeter (4.5 points)

1.1 — 0.5 pt

$P\ (J\ h^{-1}) = 5.768 \times 10^{-4} AE$

$(5.710 \times 10^{-4}$ to $5.826 \times 10^{-4} AE)$

Note: Answers within ±1% of this value are acceptable.

1.2 — 1.0 pt

$P\ (J\ h^{-1} m^{-2}) = \frac{1.442 \times 10^{-4} AE}{\pi r^2}$ *or* $\frac{4.590 \times 10^{-5} AE}{r^2}$

$(1.428 \times 10^{-4}$ to $1.456 \times 10^{-4}) AE/\pi r^2$ (if with π)

$(4.544 \times 10^{-5}$ to $4.636 \times 10^{-5}) AE/r^2$ (if no π)

Note: Answers within ±1% of this value are acceptable.

1.3 — 1.5 pt

DR = 489.1 $Gy\ h^{-1}$ (1.0 pt);

$(484.2$ to $494.0)\ Gy\ h^{-1}$

D = 81.5 Gy (0.5 pt);

$(80.70$ to $82.33)\ Gy$

Note: Answers within ±1% of this value are acceptable.

1.4 — 0.5 pt

0.997 $\mu mol\ J^{-1}$

$(0.988$ to $1.007)\ \mu mol\ J^{-1}$

Note: Answers within ±1% of this value are acceptable.

1.5 — 1.0 pt (0.2 pt each)

Absorbed dose	No. of photon interactions
10	2.081×10^{11} $(2.060$ to $2.101) \times 10^{11}$
20	4.161×10^{11} $(4.119$ to $4.203) \times 10^{11}$
30	6.242×10^{11} $(6.179$ to $6.304) \times 10^{11}$

40	8.322×10^{11} $(8.239 \text{ to } 8.405) \times 10^{11}$
50	1.040×10^{12} $(1.030 \text{ to } 1.051) \times 10^{12}$

Note: Answers within ±1% of this value are acceptable.

Part 2. Egg Hatch and Dose in SIT (3.5 points)

2.1	11.60 Gy $(11.48 \text{ to } 11.71)\ Gy$ *Note: Answers within ±1% of this value are acceptable.*	0.5 pt
2.2	58.94 min or 58 min & 56 sec. $(58.4 \text{ to } 59.5)\ min$ or $(58\ min\ 21\ sec \text{ to } 59\ min\ 32\ sec)$ *Note: Answers within ±1% of this value are acceptable.*	1.0 pt
2.3	0.896 cm $(0.887 \text{ to } 0.905)\ cm$ *Note: Answers within ±1% of this value are acceptable.*	1.0 pt
2.4 (a)	0.05343 cm^2/g $(0.0529 \text{ to } 0.0540)\ cm^2/g$ *Note: Answers within ±1% of this value are acceptable.*	0.5 pt
2.4 (b)	14 sheets	0.5 pt

Part 3. Competitiveness and Dose in SIT (2.0 points)

3.1	0.256 **(0.253 to 0.258)** ***Note: Answers within ±1% of this value are acceptable.***	1.0 pt

3.2	20.21 Gy **(20.01 to 20.41) *Gy*** ***Note: Answers within ±1% of this value are acceptable.***	1.0 pt

Q3 PARTICLES FOR DESTROYING CANCER (10 pts)

Important Notes:

1. Do NOT round off figures during calculations. Round off final answers to three significant figures.
2. Express final answers in SI units.
3. Use scientific notations to express very small or very large values.
4. Refer to Sheet G2 for useful equations, constants & conversion factors.

Part 1. Proton Stopping Power, Range, and Dose (5.0 pts)

High-energy particles entering a medium interact with matter and lose energy to varying extents depending on the type of particle, energy, charge, and mechanism of interaction. These interactions gradually slow and eventually stop the particles. The ability of a material to stop a charged particle is called the linear stopping power, S.

1.1	Given the expressions below for S and the linear energy transfer LET, what is the fundamental difference between S and LET? $S = -dE/dx$ (1) $LET = dE/dx$ (2) Note that E is the particle energy and the x is the distance travelled by the particle.	**0.4 pt**

The stopping powers of photons and charged particles vary due to their different interactions with matter. Understanding the mechanisms by which radiation interacts with matter is essential for understanding how dose is deposited in a target material, which has significant implications in medical physics and radiation therapy.

Current clinical practice for use of external beam radiation in cancer treatment includes photon and electron therapy. However, photon therapy delivers excess dose to healthy tissues both proximal and distal to the treatment site. In contrast, electron therapy results in a minimal dose beyond the treatment site, but excessive multiple Coulomb scattering restricts its use to areas within 6 cm of the target surface. Protons, being mass particles, slow down faster than photons and deposit more energy as they slow down, culminating in a dose peak known as the Bragg peak.

The graph below shows the different energy deposition of photons and charged particles based on their stopping powers as they traverse through matter

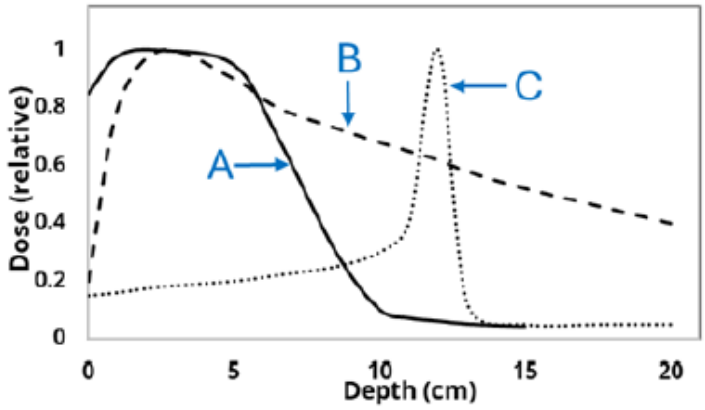


Figure 1. *Depth-dose distribution of different radiation treatments in water*

1.2	Identify the corresponding curve in **Figure 1** that represents the dose-depth distribution for electrons $(20\ \mathrm{MeV})$, photons $(18\ \ \mathrm{MeV})$, and protons $(130\ \mathrm{MeV})$ in matter (e.g., water).	**0.6 pt**

Understanding stopping power can generate insight on a new way to fight cancer. The mass stopping power is the linear stopping power divided by the material density ρ, S/ρ (in $\mathrm{J}\cdot\mathrm{m}^2\cdot\mathrm{kg}^{-1}$) and can be measured or calculated for a given material. The calculation of mass stopping power and range R of protons plays an important role in proton radiation therapy.

The Bethe-Bloch formula describes the energy loss of charged particles (such as protons) as they traverse through a material. Specifically, it describes the mass stopping power of a material for a charged particle. The Bethe-Bloch formula for the mass stopping power of protons at 1-200 MeV energy range is given in Equation 3. The combination of the $1/{\beta_v}^2$ and the logarithmic terms explain the origin of the Bragg peak.

$$\frac{S}{\rho} = -\frac{dE}{\rho dx} = \frac{5.08\times10^{-31} z^2 n}{\rho {\beta_v}^2}\left[F\left(\beta_v\right) - \ln\ \left(I\right)\right] \tag{3}$$

$$F\left(\beta_v\right) = \ln\frac{1.02\times10^6 {\beta_v}^2}{1-{\beta_v}^2} - {\beta_v}^2 \tag{4}$$

$$n = \frac{N_A Z \rho}{A} \tag{5}$$

where β_v is the velocity of the incident particle v relative to the speed of light c; I is the excitation energy of a target material ($I_{water} = 74.6$ eV); z is the charge of the incident particle ($z_{proton} = +1$); and n is the number of electrons in a material per unit volume (in m^3), calculated in terms of Avogadro's number (N_A), material density ρ_{water}, the material atomic number ($Z_{water} = 10$), and the material mass number ($A_{water} = 18$).

1.3	Simplify equation (3) for the mass stopping power S/ρ of water for protons.	**0.5 pt**

1.4	Using the derived equation from 1.3, calculate the following at kinetic energies E_K of 1, 10, and 100 MeV, assuming that protons are moving at relativistic speeds:	
	(a) proton velocity	**0.75 pt**
	(b) mass stopping power of water	**0.75 pt**
	(c) Bragg-Kleeman range R (in cm) in water as a function of the incident energy E_K. Note: $R = N_R \times {E_K}^{\beta_e}$ where $N_R = 0.0023$ g cm^{-2} MeV^{-1}, $\beta_e = 1.75$	**0.75 pt**
	(d) What general relationship can you derive between proton energy, mass stopping power, and range? Create a diagram by plotting the obtained values.	**0.5 pt**

The physical dose D at a point in a radiation field is the average energy absorbed E_{abs} per unit target mass m. The equation (6) relating dose to mass stopping power is the starting point of most proton beam line design programs. The proton fluence Φ, which is defined as the number of protons N_P per unit area A, can be used to compute the dose as shown below:

$$D \equiv \frac{E_{abs}}{m} = \frac{(dE/dx)\times dx \times N_P}{\rho \times A \times dx} = \Phi \frac{dE}{\rho dx} \qquad (6)$$

The distance protons travel past a marker position is the residual range R_{res}. With simplifying approximations that neglect the influence of nuclear reactions and Coulomb scattering, it is assumed that (1) the proton energy loss and the energy absorbed by the target material are proportional, and (2) based on the continuous slowing down approximation (CSDA), the rate of energy loss is continuous with distance (range) as the proton decelerates.

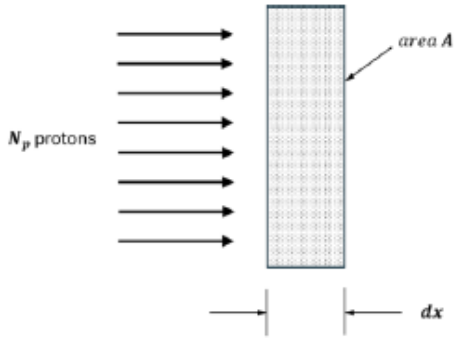


Figure 2. *Proton beams impinging on a target.*

Table 1. *Proton Stopping Power and Range (in water, liquid)*

Kinetic Energy (MeV)	S, total (MeV cm^2 g^{-1})	Range, CSDA (g cm^{-2})
5.000E+01	1.245E+01	2.227E+00
5.500E+01	1.154E+01	2.644E+00
6.000E+01	1.078E+01	3.093E+00
6.500E+01	1.013E+01	3.572E+00
7.000E+01	9.559E+00	4.080E+00
7.500E+01	9.063E+00	4.618E+00
8.000E+01	8.625E+00	5.184E+00
8.500E+01	8.236E+00	5.777E+00
9.000E+01	7.888E+00	6.398E+00
9.500E+01	7.573E+00	7.045E+00
1.000E+02	7.289E+00	7.718E+00

1.5 A proton beam of $100\ \text{MeV}$ and fluence of $1 \times 10^9\ \text{protons cm}^{-2}$ is incident on water that is $5.491\ \text{cm}$ thick in the beam direction. Ignoring nuclear reactions, what is the average dose to the water $\left(\rho_{water} = 1\ \text{g cm}^{-3}\right)$? For $100\ \text{MeV}$ protons in water, $R_{CSDA} = 7.718\ \text{g cm}^{-2}$. Note that $R_{res} = R_{CSDA} - thickness$ and that R_{CSDA} has a corresponding proton kinetic energy, which can be found in **Table 1**. Use this information to solve for proton absorbbed dose. **0.75 pt**

Part 2. Proton Therapy (5.0 pts)

Protons lose energy and deposit most of their dose over a short distance (depth) in a pattern determined by the beam energy and the target medium. By adjusting the energy, physicians can precisely control when and where the protons release most of the energy. Combining proton beams of different energies creates a Spread-Out Bragg Peak (SOBP) with a plateau-like dose that can damage a tumor across its full depth (**Figure 3**).

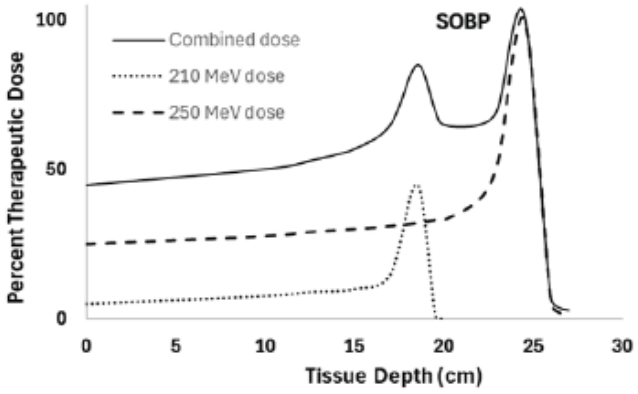


***Figure 3**. Percent therapeutic dose deposited in tissue by proton beams. The combined dose is offset for illustration only.*

2.1 **(a)** Using an arrow, indicate the specific region in the graph where the highest LET occurs. **0.3 pt**

(b) (True or False) As the proton slows down, the rate of energy loss is proportional to the square of the particle charge and the square of velocity. **0.3 pt**

(c) (True or False) In proton therapy, the exit dose is less of a concern compared to photon therapy. **0.3 pt**

(d) To treat a tumor at a depth of $18\ \text{cm}$ to $25\ \text{cm}$, which proton energy would a physician likely add to the SOBP in **Figure 3**? **0.3 pt**

A. $260\ \text{MeV}$ B. $210\ \text{MeV}$ C. $200\ \text{MeV}$ D. $235\ \text{MeV}$

The amount of radiation (dose) to cause a biological effect is measured in terms of relative biological effectiveness (RBE), which compares the dose required to produce the same biological effect between a radiation beam (often charged particles) and a reference radiation (photon), usually defined as $250\ \mathrm{keV}$ X-rays or ^{60}Co γ-rays. For the same radiation dose, high LET radiation (e.g., alpha particles, protons) will cause more damage than low LET radiation (e.g., X-rays). Radiation doses are considered isoeffective if the dose of a treatment carried out under reference conditions will produce the same clinical effects on the target. The current clinical guideline for proton therapy uses a constant RBE value $(RBE = 1.1)$ relative to high-energy photons, which accounts for the higher biological effectiveness of protons (p) over photons (x).

Several models developed for predicting RBE all rely on the linear quadratic (LQ) model of cell survival, which is one of the key tools in radiation biology and physics.

$$S\Big(D\Big) = e^{(-\alpha D+\beta D^2)} \tag{7}$$

The LQ model has been used extensively to analyze and predict responses to ionizing radiation both in vitro and in vivo. In this model, $S(D)$ is the fraction of cells that survive a delivered dose (D). The main parameters of this model, α and β, indicate how resistant a cell type is to radiation damage: cells with a higher (α/β) (~ 10) are less sensitive to a high dose per fraction.

In the LQ model (equation 7), if a proton absorbed dose D_{p} and a photon dose D_{x} are isoeffective, the relationship is expressed as:

$$\alpha_p D_p + \beta_p D_p^2 = \alpha_x D_x + \beta_x D_x^2 \tag{8}$$

$$\beta_x D_x^2 + \alpha_x D_x - \alpha_p D_p - \beta_p D_p^2 = 0 \tag{9}$$

2.2 Derive an LQ model based on equation (9) to solve for positive value of RBE *or* $\left(\frac{D_x}{D_p}\right)$. Express applicable terms as $\left(\frac{\alpha}{\beta}\right)_x$, RBE_{max}, or RBE_{min} using the following expressions: $RBE_{max} = \frac{\alpha_p}{\alpha_x}$ and $RBE_{min} = \sqrt{\frac{\beta_p}{\beta_x}}$. **1.0 pt**

RBE depends on the type of particle, the dose-averaged linear energy transfer (LET_{d}), the cell or tissue type defined by $(\alpha/\beta)_x$, and the dose per fraction (D_{p}).

Table 2. *Fit parameters*

Parameter	Values
$RBE_{max} = p_0 + \frac{p_1}{(\alpha/\beta)_x} LET_d$	$p_0 = 0.999064$
	$p_1 = 0.35605 \text{ Gy (keV µm)}^{-1}$
	$p_2 = 1.1012$
$RBE_{min} = p_2 + p_3 \sqrt{\left(\frac{\alpha}{\beta}\right)_x} LET_d$	$p_3 = 0.0038703 \text{ Gy}^{-\frac{1}{2}} \text{ (keV µm)}^{-1}$

Table 3. *Cell lines with corresponding* $(\alpha/\beta)_x$

Cell line	$(\alpha/\beta)_x$
HaCat	15.0
SKMel	3.0

2.3 **(a)** Using the LQ model derived in 2.2 and the fit parameters in **Table 2**, calculate the RBE for the two cell lines in **Table 3** as a function of proton dose $(D_p = 2 \text{ Gy})$ and LET_d $\left(1.9\ ,\ 2.5\ ,\ 4.5 \text{ keV µm}^{-1}\right)$. **1.2 pt**

(b) How does the model predict the RBE of protons in terms of the LET_d and cell radiosensitivity $(\alpha/\beta)_x$? **0.4 pt**

(c) Calculate the isoeffective photon dose D_x from the obtained *RBE* values fat $D_P = 2 \text{ Gy}$ **1.2 pt**

Question 3 Scoring Guide

Q3 PARTICLES FOR DESTROYING CANCER (10 pts)

Part 1. Proton Stopping Power, Range, and Dose (5.0 pts)

1.1	Stopping power is defined as the rate of energy loss per unit path length by a particle traveling through a material.	0.2 pt
	LET is the average energy deposited/transferred to the material per unit path length.	0.2 pt

1.2	A: Electron	0.2 pt
	B: Photon	0.2 pt
	C: Proton	0.2 pt

1.3	$\frac{S}{\rho} = -\frac{dE}{\rho dx} = \frac{0.170}{\rho {\beta_v}^2}[F(\beta_v) - 4.31]$	0.5 pt
	Other acceptable answers: $\frac{S}{\rho} = -\frac{dE}{\rho dx} = \frac{0.17}{\rho {\beta_v}^2}[F(\beta_v) - 4.31]$ $\frac{S}{\rho} = -\frac{dE}{\rho dx} = \frac{0.17}{(g \cdot cm^{-3}) {\beta_v}^2}[F(\beta_v) - 4.31]$	

1.4 (a)	$1\ MeV$: $0.046\ c = 1.38 \times 10^7\ m \cdot s^{-1}$	0.25 pt
	$10\ MeV$: $0.145\ c = 4.34 \times 10^7\ m \cdot s^{-1}$	0.25 pt
	$100\ MeV$: $0.428\ c = 1.28 \times 10^8\ m \cdot s^{-1}$	0.25 pt

1.4 (b)	$1\ MeV$: $\frac{S}{\rho} = -\frac{dE}{\rho dx} = 269\ MeV \cdot cm^2 \cdot g^{-1}$	0.25 pt
	$10\ MeV$: $\frac{S}{\rho} = -\frac{dE}{\rho dx} = 45.9\ MeV \cdot cm^2 \cdot g^{-1}$	0.25 pt

$100\ MeV$: $\frac{S}{\rho} = -\frac{dE}{\rho dx} = 7.28\ MeV \cdot\ cm^2\ \cdot g^{-1}$ — 0.25 pt

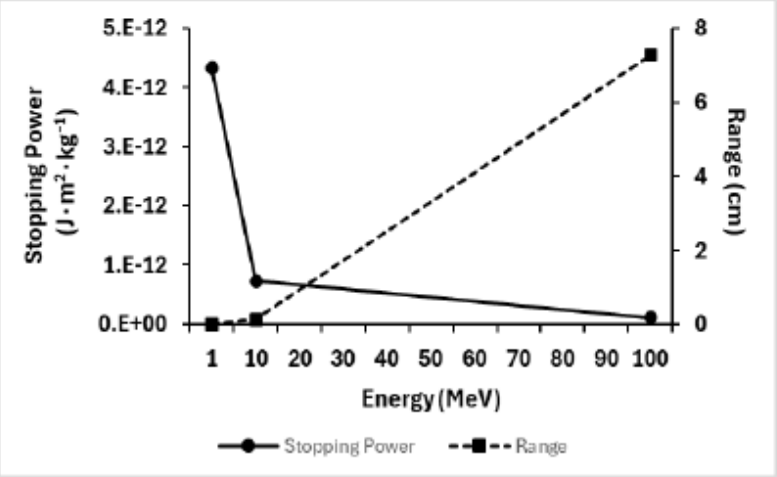
Other acceptable answers:
$1\ MeV$: $4.32 \times 10^{-12}\ J \cdot m^2 \cdot kg^{-1}$
$10\ MeV$: $7.35 \times 10^{-13}\ J \cdot m^2 \cdot kg^{-1}$
$100\ MeV$: $1.17 \times 10^{-13}\ J \cdot m^2 \cdot kg^{-1}$

1.4 (c)

$1\ MeV$: $0.00230\ or\ 2.30 \times 10^{-3}\ cm$ — 0.25 pt
$10\ MeV$: $0.129\ or\ or\ 1.29 \times 10^{-1}\ cm$ — 0.25 pt
$100\ MeV$: $7.27\ cm$ — 0.25 pt

Other acceptable answers:
$1\ MeV$: $R = 0.00230\ or\ 2.30 \times 10^{-3}\ g \cdot cm^{-2}$
$10\ MeV$: $R = 0.129\ or\ or\ 1.29 \times 10^{-1}\ g \cdot cm^{-2}$
$100\ MeV$: $R = 7.27\ g \cdot cm^{-2}$

1.4 (d) — 0.3 pt



Note: ACCEPT answer even if:
(1) Unit of Energy is in joules (J).
(2) Energy (1,10,100 MeV) is not graphed to the appropriate scale.
(3) Log_{10} of energy values is used in the graph.
(4) Obtained values (stopping power & range) are graphed with proton energy separately.

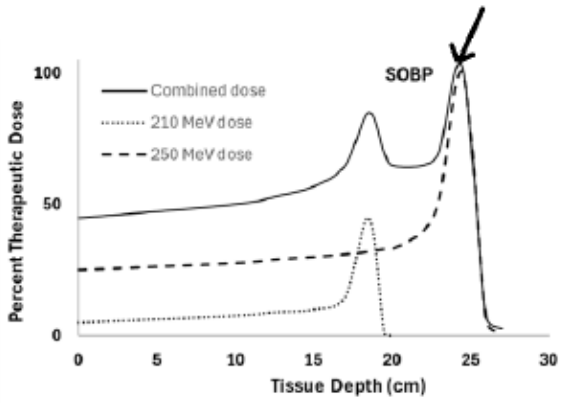
0.2 pt

The mass stopping power decreases, whereas the range of proton increases with increasing proton energy.	

1.5	$D = 1.46\ \frac{J}{kg}\ (or\ Gy)$ Other acceptable answer: $D = 9.11\ \times 10^9\ \frac{MeV}{g}$	0.75 pt

Part 2. Proton Therapy (4.0 pts)

2.1 (a) 0.3 pt



2.1 (b)	FALSE	0.3 pt

2.1 (c)	TRUE	0.3 pt

2.1 (d)	D or D. 235 MeV	0.3 pt

2.2	$RBE = \frac{D_x}{D_p} = \frac{\sqrt{\left(\frac{\alpha}{\beta}\right)_x^2 + 4D_p\left(\frac{\alpha}{\beta}\right)_x RBE_{max} + 4D_p^2 RBE_{min}^2} - \left(\frac{\alpha}{\beta}\right)_x}{2D_p}$	1.0 pt

2.3 (a)

HaCat

LET_d	RBE_{2Gy}
1.9	**1.06**
2.5	**1.08**
4.5	**1.12**

0.6 pt (0.2 pt each)

SKMel

LET_d	RBE_{2Gy}
1.9	**1.16**
2.5	**1.19**
4.5	**1.29**

0.6 pt (0.2 pt each)

2.3 (b)	RBE increases with increasing LET_d. Higher (or lower) RBE values can be expected for tissues with low (or high) $(\alpha/\beta)_x$ given similar LET_d values. Note: Exact word usage not required. Accept answers with similar interpretation.	0.2 pt 0.2 pt

2.3 (c) 0.6 pt (0.2 pt each)

HaCat

LET_d	$D_x(Gy)$
1.9	**2.13**
2.5	**2.15**
4.5	**2.24**

SKMel 0.6 pt (0.2 pt each)

LET_d	$D_x(Gy)$
1.9	**2.32**
2.5	**2.38**
4.5	**2.57**

Question 4: Mass Abundance of Isotopes

Points: 50

Time: 5.0 Hours

Q4 MASS ABUNDANCE OF ISOTOPES (10 pts)

The equivalence between mass and energy, as suggested by Einstein's Special Theory of Relativity, has a profound effect on the field of nuclear science and technology. Because the binding energy of the nucleons inside an atomic nucleus is very high (typically in the range of MeV as compared to the electron binding energy to the nucleus which is in eV or keV range), the energy that appears as the nuclear binding energy is a considerable fraction of the energy which appears as the nuclear mass. Therefore, the mass of a nucleus ${}^{A}_{Z}X_{N}$ is not simply equal to $(Zm_p + Nm_n)$ where m_p and m_n are proton mass and neutron mass, respectively. This fact makes the accurate theoretical calculation of the atomic or nuclear masses impossible, particularly for nuclides with higher mass numbers. Hence, the precise finding of atomic masses relies on experimental methods. The typical experimental apparatus used to determine the nuclear masses and relative abundances of isotopes in samples is either a mass spectrograph or mass spectrometer, which are based on the same principle of operation. A simplified schematic diagram, of cross-sectional view, showing the main components of a mass spectrograph is shown in **Figure 1**.

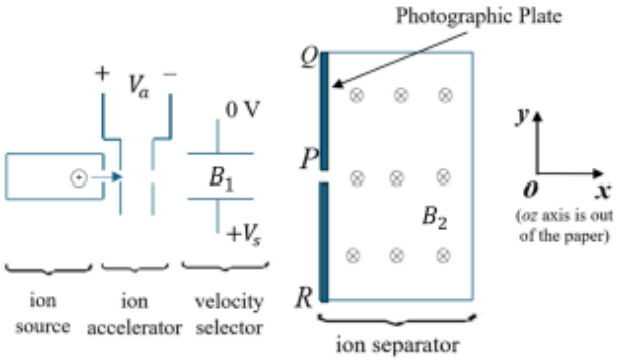


***Figure 1**. Schematic diagram of mass spectrograph*

A beam of ionized atoms is generated in the ion source. The generated ions normally have a broad distribution of velocity. These ions are accelerated to a higher energy by passing them through a potential difference in the ion accelerator. Inside the velocity selector, an electric field and a uniform magnetic field (B_1) are applied so only ions with a definite velocity enter the ion separator. Ions then make their way into the ion separator in which they travel under the influence of a second uniform magnetic field (B_2). When ions hit the photographic plate, an image is produced at the point of incidence. By measuring the position of the image, the mass of the ion can be inferred. The regions through which the ions are traveling are maintained at high vacuum conditions.

Part 1. Basic Operation of a Mass Spectrograph (2.1 pts)

For the calibration of a particular mass spectrograph, a beam of ^{12}C isotope for which the mass is taken to be exactly $12.000000\,\mathrm{u}$ on the atomic mass scale is used. Assume that singly ionized carbon ions are generated in the ion source.

1.1	If $V_a = 1.20\ \mathrm{kV}$, calculate the increase in kinetic energy (in joules) of an ion after passing through the ion accelerator.	**0.3 pt**

1.2	For an ion that exits the ion source with almost zero speed, what is the velocity at the end of the ion accelerator? You may disregard the mass of an electron compared to the atomic mass of ^{12}C.	**0.4 pt**

Electric and magnetic fields in the velocity selector are adjusted so that ions traveling along a straight line and entering the ion separator would have a velocity of $140.00\ \mathrm{km/s}$.

1.3	**(a)** What should be the direction of the magnetic field B_1 (ox, oy, oz, $-ox$, $-oy$ or $-oz$)?	**0.3 pt**
	(b) If $V_s = 400.0\ \mathrm{V}$ and the distance between the plates of the velocity selector is $2.0\ \mathrm{cm}$, calculate the magnetic flux density (in tesla) of B_1.	**0.7 pt**
	(c) If the ^{12}C is doubly ionized, what is their velocity after passing through the velocity selector?	**0.4 pt**

Part 2. Motion of Ions Inside the Ion Separator (2.9 pts)

In **Figure 1**, it is given that $PQ = PR = 148\ \mathrm{cm}$, and the magnetic flux density of B_2 is $0.030\ \mathrm{T}$. Consider an ion entering the ion separator through the hole at P with the same velocity as given in part 1.3 , i.e. $140.0\ \mathrm{km/s}$.

2.1	For a singly ionized atom, find the distance (in cm) to the point where the ion hits the photographic plate from point P.	**0.7 pt**

2.2	The resolution of the distance measurements for this mass spectrograph is $1\ \mathrm{mm}$. What is the minimum atomic mass difference $(\Delta m)_{min}$ (in u) between two elements or isotopes that can be resolved using this spectrograph? Express your answer in units of u. Assume that the uncertainties related to electric and magnetic fields are negligible.	**1.0 pt**

2.3	Assume that the ion beam produced in the ion source contains **both** singly and doubly ionized ^{12}C atoms. Draw the paths of these ions, identifying each separately, in the given Figure in the answer sheet. For each ion, indicate the point of incidence as accurately as possible according to the scale provided in the answer sheet.	**1.2 pts**

Part 3: Mass Spectrometers (1.6 pts)

The range of atomic (or nuclear) masses that can be analyzed with a mass spectrograph, for a particular value of the magnetic field strength of B_2, is not very large. In addition, it is not possible to quantitatively estimate the abundance of different elements or isotopes of a given element from the information extracted from the photographic plate. These drawbacks present in mass spectrographs can be eliminated by replacing the photographic plate in the ion separator with an electric current measuring detector installed at an exit slit at a particular point on the focal plane of the apparatus as shown in **Figure 2**. Note that all other parts are the same as given in Part 1. This set up is known as the *mass spectrometer*.

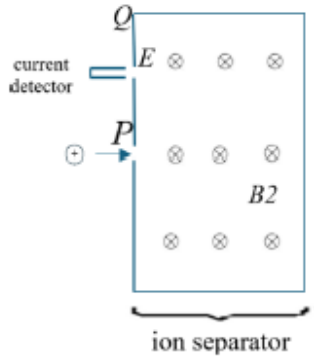


***Figure 2**. Ion separator for a mass spectrometer*

Unlike in the mass spectrograph, the flux density of the magnetic field B_2 is not constant here. The entering ion beam into the ion separator is focused onto a very narrow exit slit (E) by changing B_2. If the entering ion beam is a mixture of different isotopes, each isotope can be focused onto the slit, one at a time. The current at the exit slit is measured using a current detector installed at the slit.

3.1	Consider a mass spectrometer for which the electric and magnetic field settings of the velocity selector are the same as given in part 1.3 (b). If the beam exit slit E is located at a distance $80.0\ \text{cm}$ from P and the flux density B_2 can be set at any value between 20.0 and $320.0\ \text{mT}$, estimate the minimum and maximum atomic masses (in u) that can be analyzed using this spectrometer. Assume that all ions are singly charged.	**1.0 pt**

3.2	When an ion beam produced from naturally occurring carbon is analyzed using the spectrometer described above, the currents measured for two stable isotopes ^{12}C and ^{13}C, when they are separately focused on the exit slit, are $16.5\ \text{mA}$ and $185\ \mu A$, respectively. Find the ratio $\frac{Abundance\ of\ ^{12}C}{Abundance\ of\ ^{13}C}$. Assume that all the ions are singly charged.	**0.6 pt**

Part 4: Determining the Age of Rock samples (3.4 pts)

In geology, determining the age of rock samples found in the Earth's environment is essential in understanding and validating the different hypotheses related to the formation of the earth, and the formation of the planetary system in general. Very often, the determination of the age of a sample requires the measurement of the ratios between the abundance of different elements or different isotopes of the same element. As illustrated in Part 3, these ratios can be found, with reasonable accuracy, using mass spectrometry techniques.

Consider a sample of rock that contains a radioactive parent element P, stable daughter element D which is created from the decay of P, and another element D_s, which is a stable isotope of D. The number of nuclei of each of these elements, as determined at the present time is indicated by t_1, are $N_P(t_1)$, $N_D(t_1)$ and $N_{D_s}(t_1)$, respectively. Also assume that the rock sample was formed at a time t_0 at which time the corresponding number of nuclei are $N_P(t_0)$, $N_D(t_0)$ and $N_{D_s}(t_0)$.

4.1 **(a)** Write down a relationship between $N_P(t_0)$, $N_D(t_0)$ and $N_D(t_1)$. **0.4 pt**

(b) If the decay constant for the process $P \rightarrow D$ is λ, and the ratio $\frac{N_D(t_1)}{N_{D_s}(t_1)}$ is expressed as $\frac{N_D(t_1)}{N_{D_s}(t_1)} = G\frac{N_P(t_1)}{N_{D_s}(t_1)} + \frac{N_D(t_0)}{N_{D_s}(t_0)}$, find the coefficient G in terms of λ, t_0 and t_1. **0.7 pt**

If many rock samples had been formed from the same source (e.g. through the condensation of smaller parts from the same gas cloud), then it can be hypothesized that all these rock samples would have had the same $\frac{N_D(t_0)}{N_{D_s}(t_0)}$ ratio. Under this condition the relationship between $\frac{N_P(t_1)}{N_{D_s}(t_1)}$ and $\frac{N_D(t_1)}{N_{D_s}(t_1)}$, measured for different rock samples, becomes a linear relationship.

Radioactive decay of ^{87}Rb (half-life $t_{1/2} = 4.8 \times 10^{10}$ y produces stable ^{87}Sr isotope. Four rock samples have been analyzed using a mass spectrometer, similar to the one described in Part 3. The measured current values for ^{87}Rb, ^{87}Sr and ^{86}Sr (a stable isotope of ^{87}Sr are given in **Table 1** You may identify ^{87}Rb, ^{87}Sr and ^{86}Sr as the elements P, $D$and D_s, respectively.

Table 1. Measured currents for the rock samples

Rock Sample	Measured Current (mA)		
	^{87}Rb	^{87}Sr	^{86}Sr
1	4.8	1.65	1.91
2	0.55	0.81	1.10
3	0.8	0.37	0.46
4	7.4	1.72	1.80

4.2 **(a)** Based on the current measurements given in **Table 1**, complete the table in the answer sheet. Indicate your answers in 3 significant figures. **0.8 pt**

(b) Using the data from Part 4.2 (a), plot an appropriate graph that would allow the determination of the age of the rock samples. Clearly indicate the variables you selected for each axis of the graph. **0.6 pt**

(c) Find the gradient of the graph you plotted in part 4.2 (b). Indicate your answer in 3 significant figures. **0.3 pt**

(d) Determine the age of the rock samples in years. **0.6 pt**

Question 4 Scoring Guide

Q4 MASS AND ABUNDANCE OF ISOTOPES (10 pts)

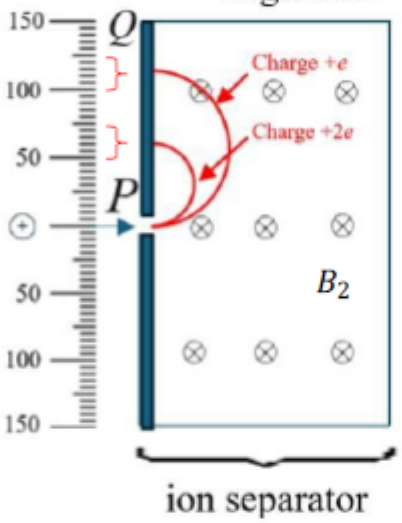
Part 1. Basic operation of a Mass Spectrograph (2.1 pts)

1.1	1.923×10^{-16} J Accept 1.91 – 1.93	0.3 pt
1.2	1.40×10^{5} m/s Accept 1.3 – 1.5	0.4 pt
1.3. (a)	oz direction	0.3 pt
1.3 (b)	0.14 T Accept 0.13 – 0.15	0.7 pt
1.3 (c)	140.00 km/s	0.4 pt

Part 2. Motion of ions inside the ion separator (3.2 pts)

2.1	116 cm	0.7 pt

2.2	0.01 u	1.0 pt

2.2	Figure 02  For correct direction for both ions For correct dimensions for both ions	 0.6 pt 0.6 pt

Part 3: Mass spectrometers (1.6 pts)

3.1	m_{min} = 5.51 u Accept 5.50-5.52 m_{max} = 88.2 u Accept 88.1 – 88.2	0.5 pt 0.5 pt

3.2	89.2 Accept 89.1 – 89.3	0.6 pt

Part 4: Determining the age of rock samples (3.4 pts)

4.1 (a) $N_P(t_1) + N_D(t_1) = N_P(t_0) + N_D(t_0)$ — 0.4 pt

4.1 (b) $G = \left[e^{\lambda(t_1 - t_0)} - 1\right]$ — 0.7 pt

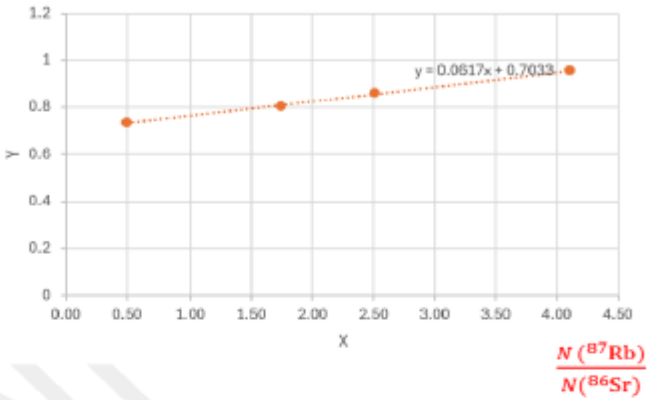
4.2 (a) 0.8 pt *(0.1 pt for each entry)*

Table 1. Nuclide ratios for the rock samples

Rock Sample	$\frac{N(^{87}\mathrm{Rb})}{N(^{86}\mathrm{Sr})}$	$\frac{N(^{87}\mathrm{Sr})}{N(^{86}\mathrm{Sr})}$
1	2.51	0.864
2	0.50	0.736
3	1.74	0.804
4	4.11	0.956

4.2 (b)

$\frac{N(^{87}Sr)}{N(^{86}Sr)}$



$\frac{N(^{87}Rb)}{N(^{86}Sr)}$

For the selection of the correct axes	0.2 pts
For the correct marking of the points (0.1/each)	0.4 pts

4.2 (c)	Gradient = 0.0617 Accept 0.061 – 0.063	0.3 pt

4.2 (d)	4.15×10^9 *years* ~ 4.2 *billion years*	0.6 pt

Question 5: Advantages of Using Low Enriched Uranium in Nuclear Reactors

Q5 ADVANTAGES OF USING LOW ENRICHED URANIUM IN NUCLEAR REACTORS (10 pts)

Light water reactors (LWRs) used for power generation are fueled with low enriched uranium (LEU) having 3% to 4% enrichment (weight % of ^{235}U), while experimental/research reactors use up to slightly less than 20% enriched uranium. Fission reactions are induced mainly in fissile ^{235}U to produce energy and neutrons to keep the reactor critical (steady-state). ^{235}U is the only naturally-occurring fissile nuclide and if all the energy is to be generated solely by fissioning of this material, the limited resources for practical fission fuel will be depleted faster. The heavier isotope of uranium, ^{238}U, also contributes to energy production through fast fission and conversion to the fissile nuclide ^{239}Pu.

In this problem, it will be studied how fissioning and conversion of ^{238}U saves ^{235}U and natural uranium resources, while also reducing the production of ^{236}U that is a long-lived radioactive waste with higher activity per unit mass. Furthermore, the problem first addresses the nuclear non-proliferation aspect of the enrichment process and concludes with the criticality safety implications of handling high enriched uranium (HEU).

In this problem, **answers should be reported to four (4) significant digits**. In addition, **no truncation** should occur in any intermediate calculations, i.e. if a result from a subproblem is used in subsequent parts, calculations should proceed without rounding off.

Part 1. Uranium Enrichment (3.5 pts)

Uranium enrichment is achieved by separating ^{235}U and ^{238}U in natural uranium, i.e. uranium having natural isotopic abundance. A typical procedure inserts uranium hexafluoride (UF_6) into a device that separates the isotopes. The device will have a feeder where mass M_F of natural uranium with enrichment x_F (in weight fraction) is inserted. The device will then produce enriched uranium with mass M_P and enrichment x_P, and depleted uranium, also called tails, with mass M_T and enrichment x_T. Intuitively, $x_P > x_F > x_T$. The material flow is depicted in **Figure 1**.

Figure 1. *Material flow in an enrichment plant.*

1.1	**(a)** Consider an ideal enrichment plant. Write the expressions for conservation of mass of uranium and just ^{235}U content.	**0.1 pt**
	(b) Derive the expression relating M_F and M_P.	**0.3 pt**

The amount of ^{235}U in the product can be calculated as follows: $M_{235} = x_P M_P$.

1.2	Assuming that $x_T = 0.002$, with $x_F = 0.72\%$ as the natural abundance of ^{235}U, express the feed mass M_F in terms of the ^{235}U mass (M_{235}) in the product for the following enrichment levels:	
	(a) 3% for a nuclear power plant fuel	**0.5 pt**
	(b) 19.7% for a research reactor fuel such as TRIGA	**0.5 pt**

The separative work unit (SWU) measures the work an enrichment plant performs to separate ^{235}U and ^{238}U isotopes. Calculating SWU is crucial for the economics of enrichment plants and nuclear fuel costs. SWU is calculated using the expression below:

$$SWU = M_P[V(x_P) - V(x_T)] - M_F[V(x_F) - V(x_T)] \quad (1)$$

where the function $V(x)$ is known as the value function, and is given by:

$$V(x) = (1 - 2x) \ \ln\left(\frac{1-x}{x}\right) \quad (2)$$

where x is the enrichment in weight fraction.

1.3	**(a)** With the same value of x_T and x_F given in part 1.2, and with $M_F = 1000$ kg, calculate for the SWU of the following values of uranium enrichment: 1.0%, 3.0%, 10.0%, 20.0%, 50.0%, and 90.0%.	**1.2 pt**
	(b) Plot x_p as a function of SWU. Indicate y-axis values. Uranium that can be weaponized requires an enrichment that is higher than 90%.	**0.6 pt**
	(c) What can be said about the separative work required to enrich uranium from a low value up to 20% compared to going higher than this threshold for commercial and civilian reactor fuel? Note: The enrichment limit is a measure to prevent the proliferation of nuclear weapons.	**0.3 pt**

Part 2. Energy from ^{235}U and ^{238}U (4.0 pts)

In LWRs, slow neutrons interacting with fissile nuclei are more likely to undergo radiative capture and fission due to their significantly higher interaction cross section (σ) compared to other interactions. ^{235}U that interacts via radiative capture is converted into radioactive ^{236}U. Radiative capture cross section (σ_γ) can be obtained from the absorption cross section (σ_a) and the fission cross section (σ_f) through the following relation: $\sigma_\gamma = \sigma_a - \sigma_f$.

Table 1. *Thermal neutron cross section data for fissile nuclides*

Fissile Nuclei	σ_a(barns)	σ_f(barns)
^{233}U	578.8	531.1
^{235}U	680.8	582.2
^{239}Pu	1011.3	742.5
^{241}Pu	1377.0	1009.0

Consider a hypothetical region in a country with a 5×10^{10} kWh annual electricity requirement. Assume only nuclear power plants supply this electricity demand and that the plants operate with an efficiency of 30%. Of the 200 MeV total energy released in a fission process, 5% is carried away by neutrinos and is not absorbed within the nuclear reactor. By approximating the atomic mass values using the mass numbers of the nuclides, the following are some important metrics if we consider only fissions occuring with ^{235}U and 365 days a year.

Table 2. *Annual Parameters for Nuclear Power Generation in Hypothetical Region (only ^{235}U as fuel)*

^{235}U probability for fission upon neutron absorption	0.8552
^{239}Pu probability for fission upon neutron absorption	0.7342
Total Net power	5708 *MW*
Mass of ^{235}U fissioned	7691 *kg*
Total Mass of ^{235}U consumed	8994 *kg*
Total mass of natural uranium used	1.631×10^6 *kg*
Total mass of ^{236}U produced	1308 *kg*

Neutrons in a reactor are also absorbed by ^{238}U along with their absorption in ^{235}U. In thermal reactors, most of the neutrons absorbed by ^{238}U convert it into ^{239}Pu, through the following reaction.

$$n + {}^{238}_{92}U \longrightarrow {}^{239}_{92}U + \gamma \xrightarrow{\beta^-} {}^{239}_{93}Np \xrightarrow{\beta^-} {}^{239}_{94}Pu \qquad (3)$$

As previously mentioned, most modern reactors use LEU fuel, primarily composed of ^{238}U. Consequently, the conversion of ^{238}U to ^{239}Pu occurs routinely during normal reactor operation. In a typical thermal reactor fueled with LEU, 20% of the fission neutrons are used to convert ^{238}U to ^{239}Pu due to resonance absorptions. In addition, out of the overall fissions, about 35% occur in plutonium and 5% in ^{238}U during the fuel's residence time in the reactor core.

2.1 Assuming that 2.6 neutrons are produced per fission, how many kilograms of plutonium are produced after one year in all the reactors that are used to generate 5×10^{10} kWh of electricity? **0.6 pt**

2.2 Assume that nuclear power plants use 3.5 wt.% enriched uranium, and the enrichment plants are operating with $x_T = 0.002$, and $x_F = 0.0072$.

(a) Calculate the amount in kilograms of ^{235}U fissioned. **0.6 pt**

(b) Calculate the amount in kilograms of ^{235}U consumed. **0.6 pt**

(c) Calculate the amount of natural uranium (in kilograms) needed to produce the required fissile mass as calculated in 2.2b. **0.8 pt**

(d) Comparing to the values in **Table 2**, provide the amount of natural uranium saved in kilograms due to contribution of ^{239}Pu and fast fission of ^{238}U. **0.2 pt**

2.3 What are the masses of ^{236}U, and ^{240}Pu present in all the reactors after one year? **1.2 pt**

Part 3. Criticality Accident (2.5 pts)

In addition to preventing the proliferation of weapons-useable material, use of LEU in commercial power reactors also ensures safety in the design by reducing the likelihood of a criticality accident. Criticality accident is the release of energy from an accidental production of a self-sustaining or divergent fission chain reaction. An example of a criticality accident that occurred with HEU is explored here.

On April 5, 1968, a criticality accident occurred at the Russian Federal Nuclear Center (VNIITF) located in the southern Ural Mountains between the cities of Ekaterinburg and Chelyabinsk. Criticality experiments have been done at VNIITF since 1957 in support of developing reactors for studying radiation tolerance of materials. One of the devices operated is the FKBN vertical lift assembly machine, with an assembly of various critical configurations of spherical metal shell with a large internal cavity surrounded by a thick reflector. This configuration allowed static and pulsed mode operations enabling research on kinetic behavior of reactor systems.

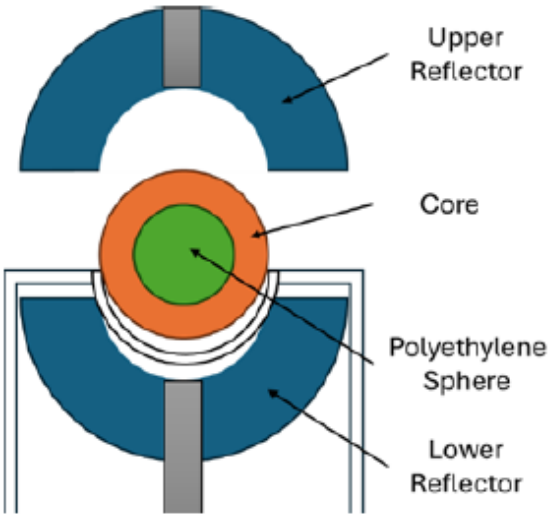


Figure 2. *Approximate accident configuration of the FKBN assembly.*

The configuration of the FBKN assembly at the time of the accident is shown in **Figure 2**. It consists of a 90% enriched (wt.%) uranium spherical metal shell core having a mass of $47.7\ \mathrm{kg}$, with an internal cavity containing a solid polyethylene sphere. The surrounding spherical reflector is made of natural uranium metal with an inside radius of $91.5\ \mathrm{mm}$ and outside radius of $200\ \mathrm{mm}$ that is split into upper and lower hemispheres. The accident occurred as the upper half of the reflector

was being lowered onto the core leading to two personnel fatalities due to the radiation burst. The excursion resulted in an estimated yield of 6×10^{16} fissions. It was identified that the incorrect position of the lower reflector was the primary cause of the accident.

3.1	What is the energy released in the accident in units of kilograms of high explosive material equivalent (1 J = 2.390×10^{-7} kg of TNT)? Assume 200 MeV is released per fission event.	**0.5 pt**

One of the victims of the criticality accident is the senior criticality safety specialist. He was located 1.7 m away from the device with an estimated accumulated neutron plus gamma dose in the range of $5 - 10$ Sv. **Table 3** shows the parameters relevant to the external dose calculation for $^{235}U(n, f)$ reaction.

Table 3. *Parameters for $^{235}U(n, f)$ reaction relevant for external dose calculation*

Parameter	Value
Average prompt gamma yield / fission	8.58
Average Gamma Ray Energy (MeV)	0.85
Photon Fluence – Dose Coefficient (pSv · cm^2) at 0.85 MeV	3.92
Average neutron yield / fission (2.0 MeV incident neutron)2.80	2.80
Prompt Neutron Average Energy (MeV)	2.0
Neutron Fluence – Dose Coefficient (pSv · cm^2) at 2.0 MeV	407

The unattenuated effective dose (E) for a point source can be approximated by:

$$E = \frac{C_D S}{4\pi r^2} \tag{4}$$

where C_D is the appropriate fluence-dose conversion coefficient, S is the source strength (particle/s), and r is the distance from the source.

3.2	Estimate the effective dose in Sv from photons for the senior specialist using point source approximation and neglecting the shielding and scattering effects of the natural uranium reflector.	**1.0 pt**

3.3	Estimate the effective dose from neutrons, and the total effective dose using point source approximation. Which ionizing radiation contributed most to the dose?	**1.0 pt**

Question 5 Scoring Guide

Q5 ADVANTAGES OF USING LOW ENRICHED URANIUM IN NUCLEAR REACTORS (10 pts)

For the boxed results containing numeric answers, three values are given: **low value**, **exact value**, **high value**. Exact value is the result obtained when computations are performed starting from the problem given without rounding-off until final answer. The low and high value corresponds to ±5% of the exact value. **Numerical results ranging from the low value up to the high value are considered correct regardless of how many significant figures are presented.** This was done to accommodate examinee solutions implementing the correct process but failed to follow the instruction not to truncate in intermediate calculations.

Part 1. Uranium Enrichment (3.5 pts)

1.1 (a)

(i) $M_F = M_p + M_T$ — 0.05 pt

(ii) $x_F M_F = x_p M_p + x_T M_T$ — 0.05 pt

1.1 (b)

$$M_F = \frac{x_T - x_p}{x_T - x_F} M_p$$

$$M_F = \frac{x_p - x_T}{x_F - x_T} M_p$$

$$M_p = \frac{x_T - x_F}{x_T - x_p} M_F$$

$$M_p = \frac{x_F - x_T}{x_p - x_T} M_F$$

0.3 pt

Any of the 4 possible answers will merit full credit (0.3 pt)

1.2 (a) — 0.5 pt

Low Value	Exact Value	High Value
170.5 M_{235}	179.5 M_{235}	188.5 M_{235}

Or

Low Value	Exact Value	High Value
170.5	179.5	188.5

1.2 (b) — 0.5 pt

Low Value	Exact Value	High Value
180.8 M_{235}	190.4 M_{235}	199.9 M_{235}

Or

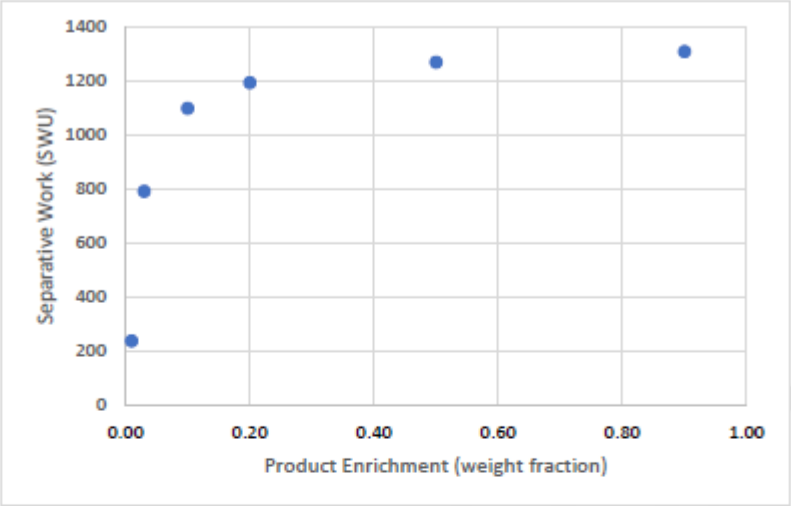
Low Value	Exact Value	High Value
180.8	190.4	199.9

1.3 (a) — 1.2 pt

0.2 pt per answer for a given x_P

x_P	SWU		
	Low Value	Exact Value	High Value
1.0%	225.4	237.3	249.2
3.0%	750.4	789.9	829.4
10.0%	1042	1097	1152
20.0%	1132	1192	1251
50.0%	1204	1268	1331
90.0%	1241	1307	1372

1.3 (b) 0.6 pt



Grading Rubric:

Case for 0.6 pt:

- All answers to problem 1.3a are correct and data points placed appropriately.

Case for 0.4 pt:

- Correct trend of steep climb from 0 to 0.2 weight fraction then gradual increase thereafter. Acceptable even without dots/markers. Acceptable even if the y-axis range is missing or incorrect.

Case for 0.2 pt:

- Examinee wrote values in y-axis regardless of the range

1.3 (c) Difficult for enrichment plants to achieve enrichment from a lower percentage up to the 20% mark. After the 20% mark, we can see that a lesser amount of work is needed to get to the higher enrichment values. 0.3 pt

Or

More work to enrich from low percentage up to 20% than increasing beyond 20% to higher enrichment level.

Grading Rubric:

Full credit is given when any of the following thoughts are expressed:

- More effort required to enrich from low value up to 0.2 weight fraction (20% enrichment)
- Less effort required to enrich above 0.2 weight fraction (20% enrichment)

Part 2. Energy from ^{235}U and ^{238}U (4.0 pts)

2.1 0.6 pt

Low Value	Exact Value	High Value
3864 kg	4068 kg	4271 kg

2.2a 0.6 pt

Low Value	Exact Value	High Value
4384 kg	4615 kg	4846 kg

2.2b 0.6 pt

Low Value	Exact Value	High Value
5127 kg	5396 kg	5666 kg

2.2c 0.8 pt

Low Value	Exact Value	High Value

9.295×10^5 kg	9.785×10^5 kg	1.027×10^6 kg

Or

Low Value	Exact Value	High Value
929,500 kg	978,500 kg	1,027,000 kg

2.2d 0.2 pt

Low Value	Exact Value	High Value
6.199×10^5 kg	6.525×10^5 kg	6.852×10^5 kg

Or

Low Value	Exact Value	High Value
619,900 kg	652,500 kg	685,200 kg

2.3 ^{236}U 0.4 pt

0.8 pt

Low Value	Exact Value	High Value
742.3 kg	781.4 kg	820.5 kg

Or

Low Value	Exact Value	High Value
745.7 kg	784.9 kg	824.2 kg

^{240}Pu

Low Value	Exact Value	High Value
941.6 kg	991.2 kg	1040.8 kg

Or

Low Value	Exact Value	High Value
945.6 kg	995.3 kg	1045.1 kg

Part 3. Criticality Accident (2.5 pts)

3.1 0.5 pt

Low Value	Exact Value	High Value
0.4365 kg TNT	0.4595 kg TNT	0.4825 kg TNT

3.2 1.0 pt

Low Value	Exact Value	High Value
5.279 Sv	5.557 Sv	5.835 Sv

3.3

$E_{neutrons}$ 0.6 pt

Low Value	Exact Value	High Value
178.9 Sv	188.3 Sv	197.7 Sv

0.3 pt

0.1 pt

E_{total}

Low Value	Exact Value	High Value
184.1 Sv	193.8 Sv	203.5 Sv

Which type of ionizing Radiation:

Neutron or Neutrons