## PREPARATORY PROBLEMS

## for the <br> First International Nuclear Science Olympiad ( $1^{\text {st }}$ INSO)

# 1st INSO Preparatory Problems 

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International Nuclear

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## PREFACE

We are pleased to present the compilation of preparatory problems for the first International Nuclear Science Olympiad (INSO). The questions included in this booklet have been carefully prepared by the INSO International Jury, a distinguished panel of experts from eight countries, organized by the International Atomic Energy Agency (IAEA).

Nuclear science plays a crucial role in advancing our understanding of the fundamental principles that govern the universe, as well as in addressing real-world challenges, from energy production to medical diagnostics. INSO aims to inspire and challenge the next generation of scientists who will contribute to the ever-evolving landscape of nuclear research.

We hope that this booklet can aid you in delving into the core principles and applications of nuclear science. Approach each question with curiosity and determination. These problems go beyond mere assessments, aiming to inspire creative thinking and a nuanced understanding of nuclear science concepts. This preparatory experience aims to both challenge and inspire, fostering a deeper appreciation for the complexities of atomic and subatomic phenomena.

We hope for your successful preparation for the competition and may your INSO experience inspire a lifelong passion for the captivating world of nuclear science.

## INSO CONTEST COMPONENTS

The contest consists of the theoretical components ( $70 \%$ weight) and experimental components ( $30 \%$ weight).

## 1. Theoretical (70\%)

1. Components: Five (5) problems, at least 3 parts within each problem. Each problem must be of one (1) hour duration.
2. Total time is 5 hours.
3. The complexity of each problem should increase gradually.
4. At least one problem should cover topics from $1,2,3$, and 4 of the syllabus.
5. An aspect of topic 6 and 7 should be incorporated within some of the problem(s).
6. No single problem solely on topic 5 .
7. Allocation of marks, marking scheme, and answer sheets will be provided.

## 2. Experimental (30\%)

1. General guidelines:
a. The experimental part consists of two sections:
i. experimental section
ii. data analysis section
b. The theoretical part of the syllabus provides the basis for all problems in the experimental part.
c. Considerations: availability of radioactive sources and detectors and relevant electronics for experimental exam.
d. Alternatively: create a scenario - students design an experiment for measurement of physical quantities.
e. Provide data - students analyze the data using graph papers/computers.
f. Conduct of simulation experiment.
g. The maximum time allocated to complete the experiment is 3.5 hours.
h. Allocation of marks, marking scheme, and answer sheets will be provided.
2. The experimental section focuses on contestant's competency in:
a. Identification of parameters that are to be measured and measurement of those parameters using provided equipment or simulation code.
b. Use of basic nuclear instruments with enough instructions provided to the contestants. Experimental observations may be from real sources or imitated sources in the laboratory. Computer simulations or codes may be used in the problems, but sufficient instructions will be provided to the contestants.
3. The data analysis section focuses on the calculation and analysis of the experimental data provided in the problems. Additional requirements are as follows:
a. Proper identification of error sources, calculation of errors, and estimation of their influence on the final results.
b. Proper use of graph papers with different scales, e.g., linear and logarithmic papers. Transformation of data to get a linear plot and finding the "Best Fit" line approximately.
c. Basic statistical analysis of observational data.
d. Knowledge of the most common experimental techniques for measuring physical quantities in nuclear science and technology.
e. Request: Instrument, simulation program, or codes for the experimental aspects

## INSO SYLLABUS

## 1. Structure of an atom and nucleus

a. The basic components of an atom
b. Basic characteristics including proton, electron, and neutron
c. The models explaining the atomic structure (evolution of models)
d. Isotopes (stable and unstable), isotones, isobars, isomers
e. The periodic table of elements
f. Properties of nucleus (size, mass, etc.)
g. Basic particle physics (quark composition of subnuclear particles)

Currently excluded: Shell model of nucleus, advanced quantum mechanics

## 2. Radiation

a. Different types of radiation (alpha, beta, gamma, $x$-ray, neutron, ionizing and nonionizing \& understanding electromagnetic spectrum)
b. Types of radioactive decay based on nucleus instability
c. Differentiate between properties of radioactive emissions (calculations of daughters, conservation of mass/energy)
d. Biological effects of radiation
e. Radiation interaction with matter (photoelectric effect, pair production, Compton scattering, etc.)
f. Dose/radiation units - Dose calculations (dose limits, shielding, etc.)
g. Radioactive decay series (parent/daughter, equilibrium, etc.)
h. Half-life, mean life, decay constants
i. Man-made vs natural sources
j. Man-made creation of radiation (x-ray production, accelerators, reactors as source, etc.)
k. Measurement of radiation (types of detectors, operating principles, etc.)

Currently excluded: Exotic forms of radiation (muons, etc.), detailed Compton scattering calculations, conservation of spin/angular momentum

## 3. Fission \& Fusion

a. Nuclear Reactions and Q-value calculations
b. Differentiate between fission and fusion reactions
c. Conversion of mass to energy $\mathrm{E}=\mathrm{MC} 2$
d. Basic relativity formulas and calculations (relating to $\mathrm{E}=\mathrm{MC} 2$ )
e. Control of fission and fusion
f. Fission and fusion as source of energy
g. Nuclear energy
h. Physics: moderation, neutron energy spectrum, scattering, cross sections, four/six factor formula, neutron life cycle
i. Engineering: design, control, components
j. Stars formation/death
k. Uranium enrichment, isotope separation

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I. Relationship to binding energy (semi-empirical mass formula, changes in binding energy)
Currently excluded: Thermodynamics of reactor operation, detailed core neutronic calculations
4. Radioactivity in the Environment
a. Natural occurrences of radioactive ores
b. Cosmic vs terrestrial sources
c. Man-made sources in the environment (fallout, etc.)
d. Radiometric dating (carbon dating, etc.)
e. Background dose calculations, normal intake or exposure of radionuclides

## 5. History of Nuclear Science

a. The historical milestones of scientists associated with the development of nuclear science and technology
b. Early applications - weapon/health (x-ray) and who discovered them
c. The IAEA establishment and role (peaceful uses)
d. History of nuclear accidents

## 6. Risk and Safety

a. Principles and concepts in radiation protection/treatment
b. Waste management principles, practices, classifications
c. Time, distance, shielding
d. ALARA - Safety and security culture
e. Emergency response (protective actions)
f. Risk communication

## 7. Applications (Energy, Health, Industry/Agriculture, Environment)

a. Nuclear application in agriculture (mutation breeding, food irradiation, sterile insect technique)
b. Health application and use as diagnostic and therapeutic treatment (x-ray radiography, computed tomography (CT), radiation therapy)
c. Industry (hydrogen production, non-destructive evaluation, crosslinking and degradation of polymers, radiation-induced reactions)
d. Sterilization
e. Radioisotope production
f. Radioactive tracing (defect detection, water tracking, etc.)
g. Nuclear power (electricity, propulsion, heat, etc.)

## Q1. CREATION OF NEW PARTICLES (10 pts)

Accelerators are modern, high precision tools with applications in a broad spectrum that ranges from material treatment, isotope production for nuclear physics and medicine, probe analysis in industry and research, to the production of high energy particle beams in physics and astronomy. The creation and study of new elementary particles is an important part of contemporary nuclear/particle physics. Especially interesting is the discovery of a very massive particle. To create a particle of mass $M$ requires energy $M c^{2}$.

1 What is the total energy $E$ of a particle of mass $m$, moving with kinetic 0.5 pts energy $K$ ?

2 What is the relativistic expression for the relationship between the total 1.0 pt energy $E$ of the incident particle and its linear momentum $p$ of the particle?

3 With enough energy, an exotic particle can be created by allowing a fastmoving particle of ordinary matter, such as a proton, to collide with a similar target particle. Let us consider a perfectly inelastic collision between two protons: an incident proton with mass $m_{p}$, kinetic energy $K$, and momentum magnitude $p$ joins with an originally stationary target proton to form a single product particle of mass $M$. By what factor should the kinetic energy of the incoming proton be increased to create a new particle with mass $3 M$ ? (Give a reasonable estimate).

4 Most modern accelerators, such as those at CERN (in Europe), at Fermi lab (near Chicago), at SLAC (at Stanford), and at DESY (in Germany), use colliding beams. Accelerators of this type are called colliders. Consider two identical colliding particles of mass $m$ and kinetic energy $K / 2$ each undergo a head-on collision to create a new particle of mass $M$ in a collider. By what factor should the total kinetic energy of the two colliding particles be increased to create a new particle with mass $3 M$ ? (Give a reasonable estimate).

Hint: before solving the problem, compare the values of the rest energy of a proton and its possible kinetic energy and speed in the collision.

## Q2. CRITICALITY OF A REACTOR (10 pts)

To make a reactor critical, it is necessary to balance the rate at which neutrons are produced within the reactor against the rate at which they disappear. Neutrons can disappear in two ways, as the result of absorption in some type of nuclear reaction, or by escaping from the surface of the reactor. When the sum of the neutron absorption and leakage rates is exactly equal to the neutron production rate, then the reactor is critical. If the production rate is greater than the sum of the absorption and leakage rates, the reactor is supercritical; conversely, if it is smaller, the reactor is subcritical. As would be expected, the production, absorption, and leakage rates depend on the size and composition of a reactor.

The multiplication factor $k$ is defined as the ratio of the rate at which neutrons are produced to the rate at which neutrons are consumed in a reactor. The neutron production rate density is given by $v \Sigma_{f} \phi$, the neutron absorption rate density is given by $\Sigma_{a} \phi$, and the neutron leakage rate density is given by $D B^{2} \phi$. Here $v$ is the average number of neutrons produced per fission, $\Sigma_{a, f}=N \sigma_{a, f}$ is the macroscopic absorption/fission cross section (with $N$ being the number of target nuclei per unit volume and $\sigma_{a, f}$ is the microscopic absorption/fission cross section), $\phi$ is the number of neutrons per unit area per unit time, $D=\frac{1}{3 \Sigma_{t r}}$ is the diffusion coefficient, $\Sigma_{t r}$ is the macroscopic transport coefficient, and $B$ is called buckling, which is equal to $\pi / R$ for a spherical geometry of radius $R$.

Nominal constants for a reactor that operates with predominantly fast neutrons are provided in Table 1.

Table 1. Nominal Constants for a Fast Reactor

| Element or <br> isotope | $\sigma_{\gamma}$ | $\sigma_{f}$ | $\sigma_{a}$ | $\sigma_{t r}$ | $v$ | $\eta$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Na | 0.0008 | 0 | 0.0008 | 3.3 | - | - |
| Al | 0.002 | 0 | 0.002 | 3.1 | - | - |
| Fe | 0.006 | 0 | 0.006 | 2.7 | - | - |
| ${ }^{235} \mathrm{U}$ | 0.25 | 1.4 | 1.65 | 6.8 | 2.6 | 2.2 |
| ${ }^{238} \mathrm{U}$ | 0.16 | 0.095 | 0.255 | 6.9 | 2.6 | 0.97 |
| ${ }^{239} \mathrm{Pu}$ | 0.26 | 1.85 | 2.11 | 6.8 | 2.98 | 2.61 |

$\mathbf{1}$ Find an expression for the multiplication factor $k$. 1.0 pt

2 Find the atomic densities of ${ }^{235} \mathrm{U}$ and ${ }^{238} \mathrm{U}$ enriched to $30 \mathrm{w} / \mathrm{o}$. The mass 2.0 pts density of the Uranium is $19.1 \mathrm{~g} / \mathrm{cm}^{3}$.

3 Suppose that fissions occur in ${ }^{235} \mathrm{U}$ only, find the value of $k$ for a $30 \mathrm{w} / \mathrm{o}$ enriched assembly of Uranium having radius of 1.0 m . What will be the value of $k$ if fissions occur in ${ }^{238} \mathrm{U}$ also?

4 Suppose $E_{R}$ is the recoverable energy per fission. Find an expression for the total energy released up to and including the $n$th generation. Find the fraction, $F_{m}$, of the energy released from a supercritical chain reaction that originates in the final $m$ generations of the chain reaction.

5 Show that for large number of generations (i.e. $n \gg 1$ ), the fraction, $F$, is approximately: $F_{m}=1-k^{-m}$. Calculate the number of fission generations required to release $99.9 \%$ of the total energy. If the mean time between generations is about $10^{-8}$ seconds, calculate the time over which $99.9 \%$ of the total energy is released.

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## Q3. ENERGY LOSS OF NEUTRON (10 pts)

In light water reactors (also called thermal reactors) most of the fissions are at thermal energies (average energy is about 0.0253 eV ). However, the neutrons produced as a result of fission are of very high energies (average energy is about 2 MeV ). In order to make the sustainable fission chain process in light water reactors, neutrons are made to make collisions with nuclei of some non-fissionable material (called moderator) in order to decrease energy of the neutrons. When a neutron is elastically scattered from a nucleus at rest, the nucleus recoils from the site of the collision. The kinetic energy of the scattered neutron is therefore smaller than the energy of the incident neutron by an amount equal to the energy acquired by the recoiling nucleus. In this way, neutrons lose energy in elastic collisions even though the internal energy of the nucleus does not change. The energy loss in elastic scattering can be found from the laws of conservation of energy and momentum. It is often beneficial to study the collision process in the Center of Mass (CM) frame of reference rather than Lab frame of reference. Consider a neutron of mass $m_{1}$, moving with velocity $v_{1}$, colliding with a stationary nucleus of mass $M_{2}$.

1 Draw diagrams to show collision process in the Laboratory (Lab) and the 1.0 pt Center of Mass (CM) frames of reference

2 Write an expression for the velocity of the center of mass of this moving 0.5 pts neutron and stationary target nucleus system. Also find the velocities of neutron and the nucleus as measured in the CM frame of reference.

3 What is the total momentum in the CM frame of reference before and 0.5 pts after the collision?

Assume that $\theta_{1}$ and $\theta_{2}$ are the scattering angles of neutron and nucleus in the Lab frame of reference respectively, while $\Theta$ be the scattering angle of neutron in the CM frame of reference.

4 Find an expression for the velocity of the scattered neutron in the lab 2.0 pts frame of reference in terms of velocities of the center of mass and velocity of neutron in the CM frame of reference.

5 Show that the ratio of scattered neutron energy to the incident neutron
3.0 pts energy is given by $\left(\frac{A^{2}+2 A \cos \Theta+1}{(1+A)^{2}}\right)$, where $A$ is the mass number of the target nucleus and $\Theta$ is the scattering angle of neutron in the CM frame of reference. What are the minimum and maximum possible values of this ratio?

6 If the energy of the incident neutron in the laboratory frame of reference is 1.0 MeV and is scattered at $45^{\circ}$, find the energy of the scattered neutron in the Laboratory frame of reference.

## Q4. MASS OF A NEUTRON STAR ( 10 pts)

We discuss the stability of large nuclei and estimate the mass of neutron stars theoretically. The binding energy $B(Z, N)$ of a nucleus consisting of $Z$ protons and $N$ neutrons is given by the Semi-empirical mass formula:

$$
\begin{equation*}
B(Z, N)=a_{v} A-a_{s} A^{\frac{2}{3}}-a_{c} \frac{Z^{2}}{A^{\frac{1}{3}}}-a_{s y m} \frac{(N-Z)^{2}}{A} \tag{1}
\end{equation*}
$$

Where $A=Z+N, a_{v}=15.8 \mathrm{MeV}$ is the coefficient of volume term, $a_{s}=17.8 \mathrm{MeV}$ is the coefficient of surface term, $a_{c}=0.711 \mathrm{MeV}$ is the coefficient of Coulomb energy term, and $a_{\text {sym }}=23.7 \mathrm{MeV}$ is the coefficient of symmetry energy term.

1 Find an expression for the atomic mass of a nucleus consisting of $Z \quad 0.5$ pts protons and $N$ neutrons as predicted by the Semi-empirical mass formula.

2 Mirror nuclei are nuclei with the same mass number $A$ and interchanged values of $N$ and $Z$. Which terms in the above formula give rise to differences in atomic mass between a pair of mirror nuclei? Calculate the predicted atomic mass difference between ${ }^{11} C(Z=6)$ and ${ }^{11} B(Z=5)$ in units of $\mathrm{MeV} / \mathrm{c}^{2}$. From this information and any general considerations, what can you deduce about the possible decay modes for these two nuclei?

3 Under the condition of $Z=N$, determine $A$ for maximizing the binding energy per nucleon, $B / A$.

4 Under the condition of fixed $A$, the atomic number of the most stable nucleus $Z^{*}$ is determined by maximizing $B(Z, A-Z)$. For $A=197$, calculate $Z^{*}$ using Eq. (1).

5 Use the semi empirical mass formula to calculate the energy released when ${ }^{238} U$ fissions symmetrically. Propose why fission products are often unstable.

6 A neutron star can be crudely approximated as a large assembly 1.0 pt ( $\geq 10^{55}$ ) of neutrons. Use the semi empirical mass formula to estimate the binding energy per nucleon of a neutron star. Comment on the physical implications of the value obtained.

7 We assume that $N=A$ and $Z=0$ is realized for sufficiently large $A$ and Eq. (1) continues to hold with the addition of the gravitational binding energy. The binding energy due to gravity is $B_{\text {grav }}=\frac{3 G M^{2}}{5 R}$, where $M=$ $m_{n} A$ and $R=R_{0} A^{\frac{1}{3}}$ with $R_{0} \approx 1.1 \times 10^{-15} \mathrm{~m}=1.1 \mathrm{fm}$ are the mass and the radius of the nucleus, respectively. For $B_{\text {grav }}=a_{\text {grav }} A^{\frac{5}{3}}$, obtain $a_{\text {grav }}$ in the MeV unit up to the first significant digit. Then, ignoring the surface term, estimate $A_{c}$ (critical value of $A$ ) up to the first significant digit. In the calculation, use $m_{N} c^{2} \simeq 939 \mathrm{MeV}$ and $G=\hbar c / M_{P}^{2}$ where $M_{P} c^{2} \simeq$ $1.22 \times 10^{22} \mathrm{MeV}$ and $\hbar c \simeq 197 \mathrm{MeV} \cdot f m . M_{p}$ is a quantity often called the Planck mass. The gravitational effect is extremely tiny as compared to the typical scale in nuclear physics and this scale difference is manifest for this expression of $G$ with $M_{p}$ in the MeV unit.

## Q5. RADIOACTIVE DATING ( 10 pts )

Many different radioactive isotopes and techniques are used for dating. All rely on the fact that certain elements (particularly uranium and potassium) contain a number of different isotopes whose half-life is exactly known and therefore the relative concentrations of these isotopes within a rock or mineral can measure the age. Either a whole rock or a single mineral grain can be dated. Some techniques place the sample in a nuclear reactor first to excite the isotopes present, and then measure these isotopes using a mass spectrometer. Others place mineral grains under a special microscope, firing a laser beam at the grains which ionizes the mineral and releases the isotopes. The isotopes are then measured within the same machine by an attached mass spectrometer.

## Part 1. Naturally Occurring Uranium ( 6.0 pts)

Naturally occurring uranium is composed of two major isotopes, Uranium-238 ( $\left.{ }_{92}^{238} \mathrm{U}\right)$ having $99.28 \%$ natural abundance), Uranium-235 ( ${ }_{92}^{235} \mathrm{U}$ ) ( $0.72 \%$ )
1.1 Find the number of atoms of U-238 for each atom of U-235. 1.0 pt
1.2 Equal amounts of each isotope existed in the Earth's crust at its 2.0 pt formation. Estimate the age of the Earth. The half-lives are: $\left({ }_{92}^{238} \mathrm{U}\right) 4.51 \mathrm{x}$ $10^{9} \mathrm{y},\left({ }_{92}^{235} \mathrm{U}\right) 7.13 \times 10^{8} \mathrm{y}$.
1.3 Since the half-life of $\left({ }_{92}^{235} \mathrm{U}\right)\left(7.13 \times 10^{8} \mathrm{y}\right)$ is less than that of $\left({ }_{92}^{238} \mathrm{U}\right) \quad 3.0 \mathrm{pt}$ ( $4.51 \times 10^{9} \mathrm{y}$ ), the isotopic abundance of $\left({ }_{92}^{235} \mathrm{U}\right)$ has been steadily decreasing since the earth was formed. How long ago was the isotopic abundance of $\left({ }_{92}^{235} \mathrm{U}\right)$ equal to $3.0 \mathrm{a} / \mathrm{o}$, the enrichment of the uranium used in many nuclear power plants?

## Part 2. Radioactive Tracer (4 pts)

Radioactive tracer technology is an important tool for measuring component wear on a realtime basis and is especially useful in measuring engine wear as it is affected by combustion system operation and lubricant performance. Combustion system operation including the use of early and/or late fuel injection and EGR for emissions control can have a profound effect on
after treatment contamination and engine reliability due to wear and tear of the engine. To facilitate wear measurement at points of interest, including ring reversal locations, direct beam irradiations are used to create distinguishable spots of radiation on the surface. The irradiated parts are then reinstalled in the test engine and the engine is run under typical or extreme operating conditions. Gamma rays emitted from radio nuclides of irradiated wear particles abrading from the rings, bearings and liner during engine operation serve as detectable tracers as the particles circulate in the lubrication system. Measuring the level of radiation associated with these particles using a gamma ray spectrometer provides a direct measure of the mass of wear particles present in the oil at the time of measurement. When compared to calibration values, these measurements give the direct amount of wear incurred during a given test period.

$$
\begin{aligned}
& \text { 2.1 A steel compression ring for the piston of a car is irradiated with neutrons } 4.0 \text { pts } \\
& \text { until it has a uniformly distributed activity of } 4.0 \times 10^{5} \mathrm{~Bq} \text { due to the } \\
& \text { formation of }{ }_{26}^{59} \mathrm{Fe} \text {. The ring is immediately installed in the engine. After } \\
& \text { the engine has been running for } 30 \text { days, a } 100 \mathrm{~cm}^{3} \text { of the engine oil is } \\
& \text { taken out and } 126 \text { disintegrations are recorded from it during a } 10 \text { min } \\
& \text { counting period. If the total volume of the oil is } 5.0 \times 10^{-3} \mathrm{~m}^{3} \text {, what } \\
& \text { fraction of the ring has worn away during the running period? Assume } \\
& \text { all the metal worn away is in suspension in the oil. (Half-life of }{ }_{26}^{59} \mathrm{Fe}=45 \\
& \text { days) }
\end{aligned}
$$

## Q6. RADIOACTIVE EQUILIBRIUM (10 pts)

The use of radioisotopes in research and stream survey work necessitates the collection of a large number of samples that may contain radioisotopes which yield radioactive daughters. When analyzing such samples, it is necessary to allow the samples to reach equilibrium. The time required to reach equilibrium can be calculated from basic nuclear physics equations.

A parent element $A$ is radioactive with decay constant $\lambda_{1}$. It decays into a radioactive daughter $B$ with decay constant $\lambda_{2}$, such that $\lambda_{2} \gg \lambda_{1}$. The element $B$ decays into a stable granddaughter element $C$. At any time $t$ the number of nuclei of $A$ is $N_{1}(t)$ and the number of nuclei of $B$ is $N_{2}(t)$. The total number of nuclei is $N_{0}$. The ratio $R=N_{2}(t) / N_{0}$ can be shown as Eq. 1.

$$
\begin{equation*}
R=\frac{N_{2}(t)}{N_{o}}=\frac{\lambda_{1}}{\lambda_{2}-\lambda_{1}}\left(e^{-\lambda_{1} t}-e^{-\lambda_{2} t}\right) \tag{1}
\end{equation*}
$$

1 Draw the plot of as a function of time. 1.0 pt

2 Obtain the expressions for variation of with when:
$2.1 t$ is very small
$2.2 t$ is very large

3 Suppose you are given sufficient experimental data of $R$ as a function of 2.0 pts time. How would you graphically determine the value of $\lambda_{2}$ and $\lambda_{1}$ ?

4 By using the equation for $R$ above, deduce the approximate relationship, $\quad 3.0$ pts

$$
N_{2} \lambda_{2}=N_{1} \lambda_{1}
$$

State the conditions under which it is valid.

5 Draw the behavior of $N_{3}(t)$ the number of $C$ nuclei, against $t . \quad 2.0$ pts
Useful relationship for small x ,

$$
e^{x}=1+x+\cdots
$$

## Q7. RADIOACTIVITY AS SOURCE OF HEAT (10 pts)

Alpha radiation is used to provide heating for spacecraft. Unlike radioisotope thermoelectric generators that convert heat to electricity, radioisotope thermal generators make direct use of the heat generated by alpha decay. The high alpha emission rate of some Polonium (Po) isotopes makes them a compact and lightweight source of energy. This makes Po-210 (with half-life of 138 days) a good energy source candidate in spacecraft/satellites where volume and weight are critical. Traditionally, Pu-238 has been used as an energy source in space exploration but other isotopes, including Po-210, have been investigated as well. There has also been interest in coupling a Po source with a thermoelectric cell to produce power for spacecraft.

1 Calculate $Q$ value of polonium alpha decay process, i.e., ${ }_{84}^{210} \mathrm{Po} \rightarrow{ }_{82}^{206} \mathrm{~Pb}+1.0 \mathrm{pt}$ ${ }_{2}^{4} \mathrm{He}+\mathrm{Q}$ given that the mass of polonium-210 is 209.98287 amu , the mass of lead is 205.97447 amu , mass of an alpha particle is 4.0026 amu and $c=3 \times 10^{8} \mathrm{~m} / \mathrm{s}$.

2 Calculate the kinetic energy of the alpha particles emitted from 3.0 pts polonium.

3 Find the amount of heat emitted by a 1.00 mg sample of Po- 210 during 3.0 pts its average lifetime.

4 What mass of ${ }_{84}^{210} \mathrm{Po}$ is required to produce 1 MW of thermal energy from 3.0 pts its radioactive decay?

## Q8. MODEL OF THE ATOM (10 pts)

## Part 1. Plum pudding model of the atom ( 4.2 pts )

One of the earliest models to explain the atomic structure was the "plum pudding" model put forth by J. J. Thomson in the year 1904. In this model, it is assumed that the total positive charge $Z e$ of the atom is uniformly distributed within a spherical volume, say with radius $a$. The total negative charge ( $-Z e$ ) is assumed to be embedded in the cloud of positive charge in the form of $Z$ number of point charges, each with a charge of $-e$. Consider the hydrogen atom in this model.
1.1 Write down an expression for the charge density of the positively 0.4 pts charged cloud.
1.2 Find an expression for the electric field intensity at a point, distance, $r \quad 0.8 \mathrm{pts}$ from the center of the atom.
1.3 Show that the electron undergoes simple harmonic motion and obtain 1.0 pt an expression for the angular frequency related to the electron.
1.4 Estimate the maximum speed the electron would have according to this 1.4 pts model. Disregard any radiative energy loss from the electron.

### 1.5 Based on your answer to the question 1.4 above, explain why the "plum 0.6 pts pudding" model is not a good model to explain the atomic structure

## Part 2. Coulomb Barrier (5.8 pts)

In the currently accepted nuclear/atomic model, nucleons are confined to the atomic nucleus while electrons are outside of the nucleus but bound to the nucleus through the electromagnetic force. It has been found that a beam of protons with kinetic energy $1.42 \times 10^{-12} \mathrm{~J}$ can barely overcome the coulomb potential barrier of an Fe nuclide $(\mathrm{Z}=26)$ and just reach the surface of the nucleus.
2.1 Assuming a spherical shape, calculate the radius of the Fe nuclide.
2.2 The radius of the $\mathrm{He}-4$ is given as 1.74 fm . Use this information to find 1.0 pt the mass number of Fe isotope used as the target.
2.3 Using the following data, calculate the binding energy per nucleon in 1.2 pts

MeV for the Fe isotope used as the target.

Mass of the Fe nucleus, $m_{F e}=9.451982 \times 10^{-26} \mathrm{~kg}$
Mass of the proton $m_{p}=1.672622 \times 10^{-27} \mathrm{~kg}$
Mass of the neutron, $m_{n}=1.674928 \times 10^{-27} \mathrm{~kg}$
2.4 Calculate the density of nuclear matter for the given Fe isotope. Give at 1.2 pts least one example of where you can find such high densities.
2.5 Write down the number of positively charged quarks and the number of 1.0 pt negatively charged quarks that can be found in this Fe isotope.

## Q9. H-3 DECAY (10 pts)

## Part 1. Radioactive decay ( 6.5 pts)

Nuclide masses of the following light nuclei are given as follows.

| Nuclide | Mass (u) |
| :--- | :--- |
| ${ }^{1} \mathrm{H}$ | 1.00783 |
| ${ }^{2} \mathrm{H}$ | 2.01410 |
| ${ }^{3} \mathrm{H}$ | 3.01605 |
| ${ }^{3} \mathrm{He}$ | 3.01603 |
| ${ }^{4} \mathrm{He}$ | 4.02603 |
| ${ }^{6} \mathrm{Li}$ | 6.01512 |

Suppose the reaction for the decay of $\mathrm{H}-3$ to $\mathrm{He}-3$ is written down as

$$
\begin{equation*}
{ }^{3} \mathrm{H} \rightarrow{ }^{3} \mathrm{He}+\mathrm{p}_{c}+\mathrm{p}_{n} \tag{1}
\end{equation*}
$$

where $p_{c}$ indicates a charged particle and $p_{n}$ indicates a neutral particle.
$\square$
1.1 Identify $p_{c}$ and $p_{n}$
$p_{n}$ :
1.2 Which particle, $p_{c}$ or $p_{n}$ is easier to detect? Give reason(s) for your 1.0 pt answer.
1.3 Calculate the maximum possible kinetic energy for the particle $p_{c} \quad 1.0 \mathrm{pt}$ assuming that the mass of $p_{n}$ is zero.
1.4 In the following graph, roughly indicate how the energy is distributed for 1.0 pt the emitted $p_{c}$ particles. Indicate at least one numerical value (excluding zero) on the horizontal axis of the graph.
$\square$
1.5 How does the energy distribution that you plotted in (1.4) above differ 1.0 pt from the energy distribution for alpha particles emitted in a radioactive decay, in general?
1.6 If the energy of the particle $p_{c}$ is found to be 16.1 keV for a particular 2.0 pts decay, what would be the energy of $p_{n}$ for this case.

## Part 2. Enriched hydrogen ( 3.5 pts)

Half-life for the radioactive decay process given in (Part 1) above is 12.33 years. An enriched sample of hydrogen which contains 0.1 g of tritium was prepared, and it is observed that this sample produces 88 J of heat per hour.

### 2.1 Find the decay constant for $\mathrm{H}-3$ decay. <br> 0.5 pts

2.2 What is the initial activity of the sample?
1.0 pt
2.3 Assuming the activity of the sample does not change very much within 2.0 pts a period of one hour, calculate the average energy of the $p_{c}$ particles emitted.

## Q10. RADIOACTIVE POWER SYSTEMS (10 pts)

## Part 1. Reaction Energy and Half-life Calculation (5.8 pts)

Energy required to run electrical/electronic systems in spacecraft is produced mainly by solar panels and by Radioactive Power Systems (RPS). In some cases where spacecraft are employed to explore regions far away from the solar system, the only viable method to generate energy required is the use of RPS. One of the elements which has the desirable properties to be used as fuel in RPS is ${ }_{94}^{238} \mathrm{Pu}$. You are provided with the following data.

| Mass of ${ }_{94}^{238} \mathrm{Pu}=238.049553 \mathrm{u}$ | Mass of ${ }_{92}^{234} \mathrm{U}=234.040950 \mathrm{u}$ |
| :--- | :--- |
| Mass of ${ }_{93}^{236} \mathrm{~Np}=236.046570 \mathrm{u}$ | Mass of alpha $=4.001506 \mathrm{u}$ |

Mass of deuteron $=2.013553 \mathrm{u}$
1.1 Calculate the Q -value for each of the following two processes.
1.1.1 Emission of an alpha particle from ${ }_{94}^{238} \mathrm{Pu}$. 1.0 pt
1.1.2 Emission of a deuteron from ${ }_{94}^{238} \mathrm{Pu}$. 1.0 pt
1.2 Why is ${ }_{94}^{238} \mathrm{Pu}$ more likely to emit $\alpha$-particles than deuterons? 0.8 pts
1.3 In the decay process of ${ }_{94}^{238} \mathrm{Pu}$, each of the $\alpha$-particle is emitted with an 1.4 pts energy of 5.5 MeV . Taking into consideration the recoil of the daughter nucleus, calculate the total kinetic energy released in each decay. You may assume that the parent nucleus is at rest.
1.4 One method of experimentally determining the half-life of ${ }^{238} \mathrm{Pu}$ is to immerse a small piece of Pu-238 in liquid nitrogen of a volume large enough to stop all emitted a-particles and measure the rate of evaporation of the liquid. In one of such experiments, the measured evaporation rate corresponded to 68.20 W , when 120 g of Pu-238 was immersed in liquid nitrogen. Use this information to calculate the halflife of Pu-238.

## Part 2. Given mass of Plutonium ( 4.2 pts)

A sample of $\mathrm{PuO}_{2}$, which contains 476 g of ${ }_{94}^{238} \mathrm{Pu}$ is used as fuel for a RPS in a certain spacecraft.
2.1 Calculate the power (energy released per unit time) generated due to 1.2 pts the decay of ${ }_{94}^{238} \mathrm{Pu}$, at the beginning.
2.2 Draw a graph to qualitatively show the variation of generated power with 0.6 pts time.

2.3 If the initially generated power is sixteen times more than the minimum
1.2 pts power requirement to operate the spacecraft, calculate the lifetime of the RPS?
2.4 What would be the activity of the fuel source at the end of the lifetime
1.2 pts of the RPS?

## Q11. RADIOISOTOPE APPLICATION (10 pts)

1. A very small sealed radioactive source of ${ }^{137} \mathrm{Cs}$ having a strength of $1 \mu \mathrm{Ci}$ is being used in a laboratory. Nuclear data for ${ }^{137} \mathrm{Cs}$ shows that $85.1 \%$ decays emit $\gamma$ radiation with energy 0.662 MeV .
1.1 Find the number of $\gamma$ rays emitted per second?
1.2 A person, inadvertently, expose to the $\gamma$ radiation emitted from this source. The period of exposure is 2 hrs . If the mass of the person is 75 kg and only $5 \%$ of the emitted $\gamma$ radiation is absorbed by the body, calculate the absorbed dose and the dose equivalent.
1.3 If the same person absorbs the same amount of energy through $\alpha$ - 0.8 pts radiation, instead of $\gamma$-radiation, what would be the absorbed dose and dose equivalent?
2. Now assume that the source described in (1) above is dropped in a laboratory with dimensions of $12 \times 12 \mathrm{~m}$. A technician wants to use a radiation survey meter (detector) to locate the source. The detector has an effective window area of $10 \mathrm{~cm}^{2}$, and an intrinsic detection efficiency of $80 \%$.
2.1 Assuming the detector is placed very close to the floor level of the laboratory and perpendicular to the $\gamma$ flux, calculate the minimum count rate expected due to $\gamma$-radiation. State whether this is an observable count rate or not.
2.2 How close the detector must be moved to the source to have a count 1.6 pts
rate of $20 \mathrm{~s}^{-1}$ ?
2.3 Based on your answers to (2.1) and (2.2) above, state whether it is a good 0.6 pts
method to locate the source using the specified detector.
3. Use of radioactive elements to monitor various industrial processes is very common. In one of such applications, a radioactive tracer is used to monitor or verify the speed of industrial sludge along underground pipes. Assume that a solid radioactive element with a known halflife of 32 min is added at a certain point to the sludge flowing through a pipeline. A detector with an overall efficiency of $25 \%$ is used to measure the activity at a point 0.5 km down the pipeline.
3.1 If it is known that flow speed of sludge varies between 0.4 and $0.6 \mathrm{~ms}^{-1}$ 2.4 pts and the minimum count rate required at the detector, to discriminate against the background, is $10 \mathrm{~s}^{-1}$, find the strength of the added radioactive source in Bq .

## Q12. NUCLEAR BINDING ENERGY (10 pts)

The stability of an atomic nucleus is a fundamental concept in nuclear physics, with implications for everything from the energy that powers stars to the synthesis of new elements. Nuclear binding energy, the energy required to disassemble a nucleus into its constituent protons and neutrons, is a key indicator of this stability. Understanding the factors that contribute to nuclear binding energy helps scientists predict the stability of isotopes and the likelihood of radioactive decay. This section explores the mathematical expressions that describe nuclear binding energy and uses empirical data to analyze the stability of various nuclides.

## Part 1. Nuclear binding energy ( 2.5 pts)

Nuclear binding energy is the energy that holds a nucleus together, overcoming the repulsive forces between protons. It is a critical factor in determining the stability and existence of nuclides.

> 1.1 Define the nuclear binding energy $B$ for a nuclide ${ }_{Z}^{A} X$ with mass number 1.0 pt $A$ and atomic number $Z$.
1.2 Show that $B$ can be expressed as $B=\left[Z m\left({ }^{1} H\right)+N m_{n}-m\left({ }^{A} X\right)\right] c^{2}$, 1.5 pts where $m\left({ }^{1} H\right)$ and $m\left({ }^{A} X\right)$ represents related atomic masses.

## Part 2. Binding energy equation ( 5.0 pts)

An approximate empirical expression for nuclear binding energy, for nuclides with an odd mass number, can be written as follows.

$$
\begin{equation*}
B(Z, A)=a_{1} A-a_{2} A^{2 / 3}-a_{3} Z^{2} A^{-1 / 3}-a_{4}(A-2 Z)^{2} A^{-1} \tag{1}
\end{equation*}
$$

Here $a_{i}$ are constants and you are given that $a_{1}=15.5 \mathrm{MeV}, a_{2}=16.8 \mathrm{MeV}, a_{3}=0.714 \mathrm{MeV}$, and $a_{4}=23.20 \mathrm{MeV}$.
2.1 Use the above equation to show that the atomic number of the most stable nuclide of a set of isobaric nuclei with mass number $A$ is given by,

$$
Z=\frac{\mathrm{A}}{2+0.0154 \mathrm{~A}^{2 / 3}}
$$

2.2 Plot a rough graph to indicate the variation of $Z$ with $A$. What is the value of the gradient of the graph for smaller values of $A$ ?

2.3 Determine which one is the most stable isotope out of the following 1.0 pt isobars with $A=125$.

$$
{ }_{49} \mathrm{In},{ }_{50} \mathrm{Sn},{ }_{51} \mathrm{Sb},{ }_{52} \mathrm{Te},{ }_{53} \mathrm{I},{ }_{54} \mathrm{Xe},{ }_{55} \mathrm{Cs},{ }_{56} \mathrm{Ba}
$$

2.4 Calculate the neutron separation energy for the nuclide that you found in question 2.3 above.

## Part 3. Nuclear radius (2.5 pts)

The electrostatic Coulomb energy is a component of the total energy within a nucleus and plays a role in its overall stability. By examining the Coulomb energy, we can infer properties of the nuclear radius and its influence on nuclear structure.
3.1 You are given that the electrostatic Coulomb energy of a uniformly charged sphere with radius $R$ and total charge $Q$ is given by $E_{C}=\frac{3 Q^{2}}{20 \pi \epsilon_{0} R}$ . Now consider the pair nuclei ${ }_{20}^{39} \mathrm{Ca}$ and ${ }_{19}^{39} \mathrm{~K}$. The difference in electrostatic Coulomb energy between these two nuclei is equal to the difference in binding energy. Use this fact to determine the constant $R_{0}$, that appear in the equation $R=R_{0} A^{1 / 3}$.

## Q13. RADIATION DETECTION (10 pts)

## Part 1. Alpha particle in a magnetic field ( 6.5 pts )

A radioactive sample of ${ }_{84}^{214} \mathrm{Po}$, which decays through $\alpha$ particle emission is placed very close to a cylindrical vacuum chamber with a particle detector plane whose cross-sectional view is shown in the figure below. There exists a uniform magnetic induction $\vec{B}$ in the direction as shown.

1.1 Write down the equation for $\alpha$-decay of ${ }_{84}^{214} \mathrm{Po}$. 0.5 pts
1.2 If the emitted $\alpha$ particles from the sample enters the chamber 0.5 pts perpendicular to the detector plane and hit the detector plane, sketch a possible path of $\alpha$ particles on the above diagram.
1.3 If the magnitude of the applied magnetic field is $B=1.2 \mathrm{~T}$ and the radius of the cylindrical chamber is 70 cm , find the maximum kinetic energy (in MeV ) of $\alpha$ particles that would be hitting the detector plane. Given that the mass of the $\alpha$ particle, $m_{\alpha}=6.645 \times 10^{-27} \mathrm{~kg}$.
1.4 It was observed that the $\alpha$ particles emitted from the polonium source 1.0 pts hits the detector plane at a distance 66.52 cm from the center of the chamber. Find the kinetic energy $E_{\alpha}$ of the emitted $\alpha$ particles.
1.5 Find the speed of the $\alpha$-particles emitted from polonium and deduce that non-relativistic mechanics can be used to study the reaction dynamics of the $\alpha$-decay of polonium.
1.6 Evaluate the total energy $E_{t}$, emitted during the escape of the $\alpha$-particle. 1.5 pts

## Part 2. Ionization chamber ( 3.5 pts)

Now suppose that the source described in (Part 1) above is placed very close to the window of an ionization chamber. A schematic is shown below.

2.1 Find the number of ion pairs created by an $\alpha$-particle entering the ion
chamber, if the energy required to produce one pair of ions in air is 34 eV . Assume that the $\alpha$-particle loses its entire energy within the ion chamber.
2.2 Calculate the saturation current generated in the ionization chamber by all the $\alpha$-particles emitted if the strength of the polonium source is $1 \mu \mathrm{Ci}$.
2.3 For the further processing of the signal from the ion chamber, it is required to have a voltage signal with a minimum peak amplitude of 200 mV . If the value of the resistor $R$ is $1 \mathrm{M} \Omega$, find the minimum gain of the voltage amplifier that must be connected across A and B.

## Q14. PHOTON INTENSITY (10 pts)

A uni-direction beam of X-ray was traveling in the direction normal to the absorbing medium as shown in Figure. State your assumptions and proceed to derive the expressions for the absorption coefficients for material A and B. For this problem, $\mathbf{I}$ is the intensity of the $X$-ray at the point just before entering the next material and $\mathbf{x}$ is the thickness of each material. Given that $I_{0}, I_{1}, I_{2}$ are measured. Assume that there is no interaction in the vacuum and that the interaction in each medium is only due to absorption.


Figure 1. Photon attenuation in materials $A$ and $B$

## 1 Calculate the absorption coefficient at each slab ( $k_{A}$ and $k_{B}$ )

2 One technique often employed on this kind of problem is to calculate for the effective attenuation coefficient. In such case, the intensity $I_{1}$ and $I_{2}$ are calculated with the effective attenuation coefficients, $k_{I}$ and $k_{I I}$ as $I_{1}=I_{0} e^{-k_{I} w_{I}}$ and $I_{2}=I_{1} e^{-k_{I I} w_{I I}}$ where $w_{I}=x_{1}+x_{2}$ and $w_{I I}=x_{4}+x_{5}$. Develop the expression for $k_{I}$ and $k_{I I}$ and describe the relation between $k_{I}$ and $k_{I I}$ and describe the relation between $k_{I}$ and $k_{I I}$ to $k_{A}$ and $k_{B}$.

3 Explain what complication that may happen if $x_{1}$ is comparable to $x_{5}$ and 4.5 pts $x_{2}$ is comparable to $x_{4}$, where as $I_{1} / I_{0}$ and $I_{2} / I_{1}$ are close to unity. How can this be fixed without resorting to reconducting the experiment?

Taylor expansion, $f(x)=f(a)+\frac{f^{\prime}(a)}{1!}(x-a)+\frac{f^{\prime \prime}(a)}{2!}(x-a)^{2}+\cdots+\frac{f^{n}(a)}{n!}(x-a)^{n}$

## Q15. NUCLEAR FISSION (10 pts)

A nuclear reaction known as nuclear fission occurs when the nucleus of an atom splits into two or more smaller fragments while releasing energy. Fission fragments are very energetic and hold kinetic energy which is shared between them and with neutrons in the reaction. Fission fragments can be found using a variety of experimental methods. Time-of-Flight (TOF) Measurement is a technique used to determine how long it takes for fission fragments to travel over a specific distance. The characteristics of fission fragments can be determined due to the time of flight is related to the mass and charge of the fragments.

Measurement of fission fragments' total kinetic energy is often made by measuring the time of flight of fission fragments. At the moment of fission, the two fission fragments split into two separate directions. Knowing the distance between the location of fission and the detector, we can use the flight time of the fragments to calculate the velocity of the compound nucleus.

$$
\begin{gather*}
1 \mathrm{MeV}=1.6 \times 10^{-13} \mathrm{~J}  \tag{1}\\
1 \mathrm{Da}=1.66 \times 10^{-27} \mathrm{~kg}=931.494 \mathrm{MeV} / \mathrm{c}^{2}  \tag{2}\\
c=3 \times 10^{8} \mathrm{~ms}^{-1} \tag{3}
\end{gather*}
$$

1 The distance to the detector from the location of fission is 50 cm , calculate
1.1 Velocity of fragment 1 when the time of flight is 53.928 ns . 0.5 pts
1.2 Velocity of fragment 2 when the time of flight is 34.453 ns .

2 If the fissioning compound nucleus is U-236, calculate the mass of the 3.0 pts two fragments.

3 Calculate the fission total kinetic energy.
1.5 pts

4 Estimate the maximum number of neutrons emitted in this particular 2.0 pts fission reaction if the Q-value is 190 MeV
[neutron mass is $m_{n}=1.008664898, D a=939.565 \mathrm{MeV} / \mathrm{c}^{2}$ ].

5 Upon splitting, the Coulomb repulsion of the two positively charged fragments propels them in opposite directions.
5.1 Wahl systematics says that the charge of the compound nucleus, $Z_{C N}$ can be used to predict from the charge of the fission fragments, $Z_{p}=A_{p} Z_{C N} / A_{C N}$. Here $A_{p}$ is the fission fragment mass and $A_{C N}$ is the mass of the compound nucleus. Calculate the charge of fission fragments ( $Z_{1}$ and $Z_{2}$ ).
5.2 The energy of the Coulomb repulsion between two point like charge particles is given by $E_{C}=1.44 Z_{1} Z_{2} / R$. Here the distance between fragments $R$ is in units of femtometer [ fm ]. When the two fragments are fully accelerated, what is the distance between the fission fragments?
5.3 The radius of a nuclei can often be approximated with the relation $R=1.2 \sqrt[3]{A}$. Assuming that the fragments are spherical in shape, calculate the distance between the fragment surface when the fragments achieve full acceleration.

## Q16. FISSION PRODUCT (Sm-157) (10 pts)

The Japanese Nuclear Data Library (JENDL) is an extensive collection of nuclear data used in a variety of nuclear and radiation physics computations. The Japan Nuclear Data Committee (JNDC), a division of the Japan Atomic Energy Agency (JAEA), is responsible for the development and maintenance of JENDL. A collection of evaluated nuclear data includes decay data, cross-section data for a range of nuclear reactions, and other relevant nuclear parameters. Applications such as reactor physics and simulation of nuclear systems also depend on these data. In JENDL-4.0, the isotope Sm-157 is a fission product yield produced at a rate of $2.73 \times 10^{-5}$ atoms per fission. Given that the reactor operates at a power density of $100 \mathrm{MW} / \mathrm{m}^{3}$ and with a flux of $7.5 \times 10^{12}$ neutrons $\mathrm{cm}^{-2} \mathrm{~s}^{-1}$. [Assume the energy produced per fission is $3.1 \times 10^{-13} \mathrm{~J}$ ]

### 1.1 Calculate the rate of fission of $\mathrm{Sm}-157$ produced and the corresponding steady state population

1.2 Sm-157 undergoes the following decay,

$$
{ }_{62}^{157} \mathrm{Sm} \underset{\beta 8.0 \mathrm{~min}}{ }{ }_{63}^{157} E u \underset{\beta 15.2 h}{ }{ }_{64}^{157} G d
$$

Calculate the steady-state population of Eu-157.
1.3 Thermal neutrons are absorbed by Gd-157, which lowers its population. 3 pts Calculate the steady state population of Gd-157 [Gd-157 thermal neutron absorption cross section is 240,000 barns].
1.4 Calculate the amount of Gd-157 produced after the reactor was shut 4 pts down for 14 days.

## Q17. $\mathrm{BF}_{3}$ NEUTRON DETECTION (10 pts)

Neutron is a subatomic particle, which has a neutral charge. It is detected by many kinds of detectors. One of them is a $\mathrm{BF}_{3}$ gas detector. The detection process occurs when neutrons bombarding the $\mathrm{BF}_{3}$ gas and all of them are absorbed by the boron nucleus, which after a short time splits into two fragments, Li-7 and alpha particles, these two fragments in turn deposit their energies into the gas to give us a signal.

$$
\begin{equation*}
\mathrm{n}+{ }^{10} \mathrm{~B} \rightarrow{ }^{7} \mathrm{Li}+\alpha \tag{1}
\end{equation*}
$$

About $94 \%$ of these reactions result in an excited state of lithium, which released this energy as a gamma radiation with energy of about 482 keV .

Suppose we have a detection setup using a $\mathrm{BF}_{3}$ detector measuring energies of Li and alpha.

| 1 | Calculate the Q-value of the nuclear reaction given by Eq.1. | 3.0 pts |
| :--- | :--- | :--- |

2 How much energy is shared between the products when the Li is in 2.0 pts ground state and when it is in excited state?

Table provides the masses of the reaction particles and fragments.
Table 1. Masses of B, Li, $n$ and alpha

| Particle | Mass (u) |
| :---: | :---: |
| B-10 | 10.01294 |
| n | 1.00866 |
| Li-7 | 7.016 |
| Alpha | 4.0026 |
| $1 \mathrm{u}=931.494 \mathrm{MeV}$ |  |

3 For both cases mentioned in number 2, find the kinetic energies of Li 3.0 pts and $\alpha$ using the information in Table .

4 Suppose that the energy of fragments was deposited fully in the detector gas and get an energy spectrum as shown in Figure. Determine the energy of peaks $\mathbf{a}$ and $\mathbf{b}$, and if you know the number of counts under peak $\mathbf{b}$ is 3000 counts, what is the number of counts under peak $\mathbf{a}$ ?


Figure 1. Spectrum of a BF3 detector

## Q18. ALARA (10 pts)

1. (1 pt) Radiation exposure can be reduced by increasing the distance from the source of radiation. This can be explained by the application of the inverse square law. The inverse square law states that for a point source of waves that can radiate uniformly in all directions from a single point. The intensity I decreases with the square of the distance, $d$, from the source.

### 1.1 In the given scenario, a technician is present in an area for a duration of 10 minutes, and the reading on the survey meter registers 5 millirem per hour ( $\mathrm{mR} / \mathrm{h}$ ). What dose of radiation does the technician receive?

1.2 The technician wants to receive no more than a 1.0 mR dose knowing the above conditions in (1.1). What is the technician's maximum allowable stay time in the area?
2. ( $3.5 \mathbf{p t s}$ ) lodine, upon entering the body, finds its storage place in the thyroid gland, where it is later released to regulate growth and metabolism. lodine-131 is utilized to visualize the thyroid through imaging when injected into the body. Additionally, in higher doses, lodine131 is employed for the treatment of thyroid cancer. This isotope has a half-life of 8.0252 days and undergoes decay through $\beta^{-}$emission to become Xenon-131.
2.1 Write an equation for the decay.
2.2 How long will it take for $95.0 \%$ of a dose of I-131 to decay?

3 Technetium-99m has a half-life of 6.01 hours. When can the patient be 1.0 pt discharged from the hospital if they can leave once $75 \%$ of the injected technetium-99m dose has decayed?

4 Based on what is known about Radon-222's primary decay method, why 0.5 pts is inhalation so dangerous?
5. (5.5 pts) To eliminate a cancerous tumor, a quantity of gamma radiation equivalent to 2.12 J must be administered over a period of 30.0 days. This radiation is to be supplied through implanted sealed capsules containing palladium-103, an isotope with a half-life of 17.0 days that emits gamma rays at 21.0 keV energy, which are fully absorbed within the tumor.

### 5.1 Determine the initial activity of the set of capsules. <br> 1.0 pts

5.2 Find the total mass of radioactive palladium contained in the sealed 1.0 pts capsules.

6 The radioactive isotope $\mathrm{Na}-24$ undergoes beta decay with a half-life of 1.5 pts 15.0 hours. A solution containing $0.05 \mu \mathrm{Ci}$ of $\mathrm{Na}-24$ is injected into a person's circulatory system. After 4.50 hours, a blood sample from the person is examined, revealing an activity level of $8.00 \mathrm{pCi} / \mathrm{cm}^{3}$. How many liters of blood does the person's body contain?

7 A sealed capsule is inserted into a patient's tumor, containing the radiopharmaceutical phosphorus-32 ( ${ }^{32} \mathrm{P}$ ), which emits electrons. The average kinetic energy of the beta particles is 700 keV , and the initial activity of the capsule is measured at 5.22 MBq . Determine the energy absorbed during a 10.0-day period, given that the half-life is 14.26 days. $[A=\lambda N]$

8 A source is producing an intensity of $456 \mathrm{R} / \mathrm{h}$ at one meter from the 1.5 pts source. What would be the distance in meter, (m) to the 100, 5, and 2 $\mathrm{mR} / \mathrm{h}$ boundaries.

## Q19. POSITRON EMISSION TOMOGRAPHY (10 pts)

In a positron emission tomography (PET) scan, a patient is injected with a positron-emitting radiopharmaceutical. The radiopharmaceutical undergoes positron decay, producing a pair of gamma-ray photons.

## Part 1. Problem Subheading (7.0 pts)

Consider a radiopharmaceutical called fluorodeoxyglucose (FDG), which contains a radioactive particle F-18 with a half-life of 110 minutes. After injection, the decay process begins, and the emitted gamma rays are detected by the PET scanner. For the questions below, assume that no other decay processes or biological processes affect the radiopharmaceutical during the given time frame.

### 1.1 At time $t=0$, a patient is injected with 10 mg of FDG. Calculate the initial 1.0 pt activity (in becquerels, Bq ) of the injected substance. (Molar mass of FDG $=181.15 \mathrm{~g} / \mathrm{mol}$ ).

1.2 After 2 hours, calculate the remaining activity of FDG in the patient's 1.0 pt body.

### 1.3 The PET scanner is operated for an hour after injecting the patient. 2.0 pts Estimate the number of FDG molecules decaying per minute during that time.

1.4 Due to the decay process, a certain amount of energy is deposited in the $\quad 3.0$ pts
patient's body. If each decay event releases 0.5 microjoules $(\mu)$ of energy,
calculate the total energy deposited in the patient's body after 4 hours.

## Part 2. Problem Subheading ( $\mathbf{3 . 0} \mathbf{~ p t s}$ )

A dynamic PET imaging study is carried out using another radiopharmaceutical with a half-life of 90 minutes. The study involves continuous imaging for 3 hours following injection. The radiopharmaceutical is injected with an initial activity of 200 megabecquerels (MBq).

From the information given, calculate:

### 2.1 The biological half-life of the radiopharmaceutical if the effective half- 2.0 pts life is 35 minutes.

2.2 In the context of nuclear medicine, what is the main advantage of using 1.0 pt short-lived radionuclides in imaging studies?

## Q20. HALF-LIFE DETERMINATION (10 pts)

## Part 1.

A student wishes to measure the half-life of a radioactive substance using a small sample. For this purpose, she uses a radiation counter of consecutive clicks which are randomly spaced in time. The counter registers 372 counts during the first ( $T$ ) interval of time, and 337 counts during the next ( $T$ ) interval. Knowing that the average background rate is 15 counts per minute.

### 1.1 Define the half-life of a radioactive source.

1.2 If the time interval $(T)$ is 5 min , calculate $\left(N_{1}\right)$ and $\left(N_{2}\right)$ where $\left(N_{1}\right)$ and 1.0 pts $\left(N_{2}\right)$ are the actual decay counts of the radioactive source in the first and second intervals respectively.
1.3 Show that the half-life of this substance can be estimated using the 3.0 pts expression:

$$
T_{\frac{1}{2}}=-T \frac{\ln 2}{\ln \left(\frac{N_{2}}{N_{1}}\right)}
$$

(Derive the expression).
1.4 Determine the half-life (in minutes) of the used radioactive source. 0.5 pts
1.5 As the counter clicks are randomly spaced in time, the counts of each 2.0 pts interval ( $T$ ) may experience an uncertainty of ( $\pm 5$ ). Estimate the absolute uncertainty in the half-life determination in part (1.3). Show your work.

## Part 2.

2.1 A radioactive detector is used to measure the count rate of a single radioactive source. It initially registered 82 counts sec-1, which dropped after 210 sec to 19 counts sec- 1 . The half-life of the substance was 70 s . Verify that the count rate does not satisfy an exponential decay law. A constant background radiation was present during the measurements. Determine its count rate.
2.2 If in (2.1) the anomaly was due to the presence of a radioactive 1.0 pt substance, with a much greater half-life than the source, how would one deduce its half-life from experimental measurements?

## Q21. FUSION REACTION (10 pts)

According to many theories, there are fission and fusion processes in the stars like the sun. The fusion process can be performed when light atoms encounter an extremely high pressure and high temperature in a very small area. Assume a deuteron and a triton are at rest when they fuse according to the reaction:

$$
\begin{equation*}
{ }_{1}^{2} \mathrm{H}+{ }_{1}^{3} \mathrm{H} \rightarrow{ }_{2}^{4} \mathrm{He}+{ }_{0}^{1} \mathrm{n} \tag{1}
\end{equation*}
$$

Knowing that: $\left[\left(m_{1}^{2} H=2.0141\right),\left(m_{3_{1} H}=3.0160\right),\left(m_{1}^{4} H e=4.0026\right)\right.$, and $\left.\left(m_{1} n=1.0087\right)\right]$.

1 How many neutrons and protons in one deuteron isotope? 0.5 pts

2 It is known that normal water consists of two hydrogen atoms and one oxygen atom. In the nuclear reactors the heavy water (consist of two deuteron atoms instead of normal hydrogen) is used as neutrons moderator. State a reason for each of the following statements:
2.1 The water which consists of deuteron is called (Heavy).
2.2 The heavy water is a better choice to moderate fast neutrons.

3 The abundance of hydrogen is $99.985 \%$ while for the deuterium is $0.015 \quad 1.0 \mathrm{pt}$ \%. Calculate the number of atoms of deuterium naturally occurring in one liter of water.

4 Referring to the conservation of momentum, show that after the 1.5 pts reaction, the ratio between the neutron velocity to the alpha particle velocity is $\left(\frac{v_{n}}{v_{\alpha}}=3.968\right)$.

5 Determine the kinetic energy (in MeV ) acquired by the neutron $\left(K E_{n}\right)$ in this reaction. What is the percentage of neutron kinetic energy to the total energy of the reaction.
$6 \quad$ In most cases, nuclear reactions are associated with relativistic effects. 4.0 pts Thus, the conservation of energy of the above reaction requires that:

$$
E_{n}+E_{\alpha}=\left(m_{n} c^{2}+K E_{n}\right)+\left(m_{\alpha} c^{2}+K E_{\alpha}\right)
$$

And the relation between the total energy, mass, and momentum for a relativistic particle is:

$$
E^{2}=P^{2} c^{2}+\left(m c^{2}\right)^{2}
$$

Use these two relations to estimate an accurate value for $\left(K E_{n}\right)$.

## Q22. SMOKE DETECTORS (10 pts)

## Part 1. How smoke detectors work (2 pts)

Smoke detectors are engineered to detect smoke and provide an alert. According to industry standards, smoke detectors should be replaced every ten years. Americium-241 (Am-241) is a radioisotope that is employed in small amoun ts within ionization type smoke detectors. Am241 is an alpha-emitter with a half-life of 432.6 years. The release of alpha particles from Am241 leads to the ionization of the air located between two-electrode plates within the ionizing chamber. A battery generates an electric potential that induces the motion of ions, resulting in the production of a modest electric current. The introduction of smoke into the chamber hinders the flow of ions, hence diminishing the conductivity of the air. This phenomenon results in a decrease in the electrical current, which initiates an alarm.

### 1.1 Provide a reason why an alpha emitting radioisotope is preferred than a beta or gamma emitting radioisotope.

1.2 Production of Am- 241 radioisotope is based on a series of nuclear 1.5 pts reactions and radioactive decay. The series begins with transmutation of $\mathrm{U}-238$ into $\mathrm{U}-239$ via neutron capture in a reactor. U-239 beta decays into Np-239, which also beta decays into Pu-239. Continued neutron irradiation of Pu-239 forms Pu-240, which also absorbs neutrons to form Pu-241. Pu-241 then decays into Am-241 via beta emission. Write the complete expression for nuclear reactions and radioactive decays in each step of the series for producing Am-241 radioisotope from U-238.
(Atomic numbers: $\mathrm{U}=92, \mathrm{~Np}=93, \mathrm{Pu}=94, \mathrm{Am}=95$ ).

## Part 2. Am-241 alternatives (1.5 pts)

Researchers have isolated 4 different alpha-emitting radioisotopes from a reactor experiment. The radioisotopes were separated, purified, and their decay constants were calculated as follows:

$$
\lambda_{A}=0.04621 d^{-1} ; \lambda_{B}=0.001899 d^{-1} ; \lambda_{C}=0.003466 d^{-1} ; \lambda_{D}=0.006931 d^{-1}
$$

### 2.1 Determine the half-life of each radioisotope. <br> 1.0 pt

### 2.2 Would any of these radioisotopes be suitable as an alternative for Am241 to be used in a smoke detector? Explain.

## Part 3. Alpha particle and its interactions ( $\mathbf{2 . 5} \mathbf{~ p t s )}$

Most smoke detectors sold today use one microcurie or less of Am-241.
3.1 How many alpha particles per second are emitted by one microcurie of 0.5 pts Am-241 in a smoke detector? Note: $1 \mathrm{Ci}=3.7 \times 10^{10} \mathrm{~Bq}$
3.2 Alpha particles emitted by Am-241 have an energy of 5.5 MeV . The 1.0 pt specific ionization or the average number of ion pairs produced per unit distance traveled by alpha particles in air is 34,000 ion pairs per cm . If 36 eV of energy has to be transferred for one ion pair to be produced in air, estimate the range of alpha particles in air.
3.3 Manufacturers of smoke detectors are required to adhere to health and safety regulations to ensure the well-being of their employees who are exposed to radiation. During a 12 -hour shift, a radiation worker receives an absorbed dose of 300 nanogray every hour from alpha radiation. Determine the equivalent dose received by the worker in the 12 - hour shift. If the exposure limit for radiation workers was set at 20 mSv , how many days can the worker report to the plant assuming that they are limited to 12 -hour shift per day? The radiation weighting factor of alpha particles is 20 .

## Part 4. Production of Am-241 (4 pts)

The production of Am-241 isotopes will depend on its half-life and the decay rate of its parent nuclide, Pu-241.

> 4.1 Write the rate equation for the buildup of $\mathrm{Am}-241$ from Pu- 241 . Sketch 0.5 pts a plot of the amount of $\mathrm{Am}-241$ and Pu- 241 as a function of time. Assume that the amount of $\mathrm{Am}-241$ is zero at $\mathrm{t}=0$.
4.2 Secular equilibrium is attained when the production rate of a 0.5 pts radioisotope is equal to its decay rate. Can secular equilibrium be attained for Am-241? Explain.
4.3 Calculate the amount of Am-241 that is formed from the decay of 50 g of pure Pu-241 for 1 day. The half-life of Pu-241 is 14.3 years while its molar mass is $241.05685 \mathrm{~g} / \mathrm{mol}$. Assume that the decay of Am-241 is negligible.
4.4 If its decay is not neglected, the number of Am-241 isotope as a function 1.0 pt of time can be obtained from the following equation:

$$
N_{\mathrm{Am}}=\frac{\lambda_{\mathrm{Pu}} N_{\mathrm{Pu}}}{\lambda_{\mathrm{Am}}-\lambda_{\mathrm{Pu}}}\left(\mathrm{e}^{-\lambda_{\mathrm{Pu}} t}-\mathrm{e}^{-\lambda_{\mathrm{Am}} t}\right)
$$

Calculate the amount of Am-241 from the decay of 50 g of pure Pu-241 for 1 day using the above equation. By how much does the result differ from your answer in (4.3)? Is it reasonable to neglect the decay of Am241 for this calculation?
4.5 How many smoke detectors, each with activity of one microcurie can be 1.0 pt produced from the amount obtained in (4.4), assuming that there is negligible loss in Am- 241 during the manufacturing process?

## Q23. RADIOACTIVE DATING (10 pts)

An important application of the fundamental radioactive decay law is in the field of radionuclide dating. The quantity of radioactive nuclides existing at a certain time, denoted as t , can be determined using the radioactive decay law:

$$
\begin{equation*}
N=N_{o} x e^{-\lambda t} \tag{1}
\end{equation*}
$$

Solving this equation for t gives

$$
\begin{equation*}
t=\frac{\ln \left(N_{0} / N\right)}{\lambda} \tag{2}
\end{equation*}
$$

In this context, the variable "t" represents the age of the artefact, which may be ascertained by considering the half-life of the radioactive nuclides, as well as the current number of radioactive nuclei present in the artefact (denoted as " N ") and the original number of radioactive nuclei ( ${ }^{(N o}{ }_{0}$ "). While the determination of N can be achieved through sample counting, a distinct methodology is required to establish the value of $\mathrm{N}_{\mathrm{o}}$.

## Part 1. Radioactive Dating (2 pts)

1.1 In the context of radioactive decay, it can be observed that the total 1.0 pt number of nuclei remains constant during the process of decay from parent $P$ to daughter $D$. This may be expressed mathematically as

$$
P_{o}=P\left(t_{o}\right)=P(t)+D(t)
$$

Assuming the absence of any daughter atoms at $t=0$, derive the equation for the age, $t$, in terms of the number of daughter, $D(t)$, and parent, $\mathrm{P}(\mathrm{t})$.
1.2 Aside from the assumption stated in \#1, give two other assumptions that 1.0 pt are required to use the derived equation for calculating the age t .

## Part 2. Naturally Occurring Radioactive Materials ( 4.5 pts)

A rock sample from Mt. Taal contains naturally occurring radioactive material, Uranium and Thorium. Natural uranium contains isotopes of U-238, U-235 and U-236, with natural abundance of U-238 of $99.27 \%$. Thorium, on the other hand, has 2 natural isotopes - Th-232 ( $99.98 \%$ abundance) and Th-230. As uranium and thorium decay at a constant rate, they emit subatomic particles and eventually turn into a stable lead. From the natural decay of Uranium and Thorium parent nuclides up to the stable isotopes, one can estimate the geological age of the rock sample by determining the ratio of the parent and stable daughter nuclide concentrations.
2.1 Given the decay chain for Th-232 below, fill in the missing isotopes and 1.5 pt determine the stable isotope of Th-232.

2.2 Find the value of $\boldsymbol{X}$ in the equation for the estimation of time, $t$, in years, $\quad 3.0 \mathrm{pts}$ given by,

$$
t=\frac{\boldsymbol{X} P b}{U+0.326 T h}
$$

where $\mathrm{Pb}, \mathrm{U}$ and Th are the concentrations of the elements of lead, uranium, and thorium in the rock sample with the following assumptions:

Common lead, $P b$, may have been present at time, $\mathrm{t}=0$, and the total lead atoms and masses from $U$ and $T h$ decay sum up to $P b_{\text {tot }}$.

$$
P b_{206}+P b_{208}+P b=P b_{t o t}
$$

the total number of atoms of stable isotopes of Pb from the U and Th is related to their decay constants $\left(\lambda_{U}=1.55 \times 10^{-10} / y r ; \lambda_{T h}=4.93 \times\right.$
$\left.10^{-11} / y r\right)$, mass, atomic weight $\left(W_{U}=238.029 \frac{g}{m o l} ; W_{T h}=\right.$ $232.0381 \frac{\mathrm{~g}}{\mathrm{~mol}}$ ) at time, t , is given by,

$$
\begin{aligned}
P b_{206} & =\frac{\text { mass of } U \times N_{A}\left(\exp \left\{\lambda_{U} t\right\}-1\right)}{\text { atomic weight of } U} \\
P b_{208} & =\frac{\text { mass of } T h \times N_{A}\left(\exp \left\{\lambda_{T h} t\right\}-1\right)}{\text { atomic weight of Th }}
\end{aligned}
$$

From Taylor expansion,

$$
e^{\lambda t}-1=\lambda t+\frac{\lambda^{2} t^{2}}{2}+\frac{\lambda^{3} t^{3}}{6}+\frac{\lambda^{4} t^{4}}{24}+\frac{\lambda^{5} t^{5}}{120}+\cdots
$$

and $\lambda_{U} t$ or $\lambda_{T h} t<1\left(\right.$ for $\left.t<6 \times 10^{9} y r s\right), e^{\lambda t}-1 \approx \lambda t$

## Part 3. Problem Subheading - Decay of Carbon-14 (3.5 pts)

One of the most significant dating methodologies involves the decay of Carbon-14 (C-14, $\mathrm{t}_{1 / 2}$ $=5730 \mathrm{y}$ ). C-14 is generated continuously by the process of neutron absorption of nitrogen14 ( $\mathrm{N}-14$ ), a reaction driven by cosmic rays in the Earth's upper atmosphere. This radioactive carbon isotope, C-14, exchanges with stable carbon isotope, C-12, inside biological systems resulting in the existence of a constant level of C -14 in living organisms. When an organism dies, the process of carbon atom exchange with the reservoir of $\mathrm{C}-14$ and its corresponding $\mathrm{C}-14$ activity will gradually diminish. The determination of an artefact's age can be achieved through the measurement of its specific activity. Once the biological material within an artefact has undergone decay for a duration equivalent to ten or more half-lives of $C$ - 14 , it is no longer possible to directly determine the C-14 activity of the artefact. In this case, one can utilize the accelerator mass spectrometry (AMS) to quantify the C-14 atoms.

> 3.1 Write the cosmic ray-induced nuclear reaction for the conversion of $\mathrm{N}-\quad 1.0 \mathrm{pt}$ 14 to $\mathrm{C}-14$.

$$
\begin{aligned}
& \text { 3.1 A sample of preserved skin from a tyrannosaurus rex (T rex) contains } 5 \\
& \mathrm{mg} \text { of carbon. AMS measurement of the sample yielded a } \mathrm{C}-14 \text { to } \mathrm{C}-12 \\
& \text { ratio of } 1.7 \times 10^{-14} \text {. How many } \mathrm{C}-14 \text { atoms are present? If the assumed } \\
& \text { constant specific activity of } \mathrm{C}-14 \text { in nature in the pre- nuclear era is } 227 \\
& \mathrm{~Bq} / \mathrm{kg} \text {, what is the age of the } \mathrm{T} \text { rex skin? Atomic weight of } \mathrm{C}=12.011 \text {; } \\
& \mathrm{t} 1 / 2 \mathrm{C}-14=5,730 \text { years. }
\end{aligned}
$$

## Q24. ELECTRON BEAM APPLICATION (10 pts)

It has been shown that gamma radiation interacts with matter to produce fast electrons, which, when absorbed in the sample, are responsible for most of the chemical changes detected. In fact, gamma irradiation can be thought of as a simple means of injecting fast electrons into a material for most purposes. Fast electrons can also be introduced directly into a sample by electron accelerators, also known as electron beam (EB) irradiators.

## Part 1. Inducing chemical reactions (5 pts)

EB irradiators could be used to induce chemical reactions in the synthesis of industrially relevant chemical reagents. An example is 2,3-butanediol, a promising chemical that could be used in various industrial applications, specifically to produce methyl ethyl ketone or MEK, an excellent liquid fuel additive, and 1,3-butadiene, raw material used to make synthetic rubber. Irradiation of liquid ethanol by electron beam yields 2,3-butanediol as one of the radiolysis products.

$$
\begin{aligned}
& \text { 1.1 The gamma irradiation of liquid ethanol also produces 2,3-butanediol as } \\
& \text { one of the products. Give one advantage of EB over gamma irradiation } \\
& \text { in liquid ethanol irradiation to produce 2,3-butanediol. }
\end{aligned}
$$

### 1.2 If an ethanol molecule becomes changed chemically when one of its 1.0 pt atoms absorbed a 10 keV gamma ray photon, how many molecules will be changed by photoelectrons for every molecule changed by the photons as such if the binding energy of the electrons in oxygen is 530 eV and the photoelectrons change ethanol at the rate of four molecules per 100 electron volts (eV).

### 1.3 A 10 g of 2,3-butanediol was prepared by irradiating ethanol in an EB facility using a $50 \mu \mathrm{~A}$ beam of 2 MeV electrons. For how many minutes would it be necessary to irradiate if all the energy could be absorbed in the ethanol, and the yield of the product could be taken to be $\mathrm{G}=2.0$ ? The $G$-value is defined as the number of molecules changed (formed or destroyed) or events per 100 eV of ionizing energy absorbed. <br> Molar mass of butanediol $=90.121 ; 1 \mathrm{~J}=6.242 \times 10^{18} \mathrm{eV}$

## Part 2. Ozone production (5 pts)

In an EB irradiation facility, ozone is produced through the interaction of oxygen in the air with energetic electrons. Ozone is toxic to all forms of life. Depending on the agency or country involved, the human threshold limit values vary from 60 to 100 parts per billion in air. Much higher concentrations can be produced inside industrial electron beam (EB) facilities, so methods for ozone removal must be provided.
2.1 In the operation of an $E B$ machine, ozone is produced in irradiated air with $G=10$. How many cubic centimeters of ozone would be produced in one minute by an accelerator delivering 10 W to air at standard T and P? Assume ideal gas behavior. Ideal gas constant $\mathrm{R}=0.0821 \mathrm{~L} \mathrm{~atm} \mathrm{~mol}^{-}$ ${ }^{1} \mathrm{~K}^{-1}$.
2.2 In the EB irradiation of ethanol, the radiation worker receives an absorbed dose of $45 \mu$ Gy every hour for every 8 -hour shift, primarily from X-rays that are produced when an electron interacts with the nucleus of a metal target.
2.2.1 Determine the equivalent dose received by the worker in the 8hour shift.
2.2.2 If the annual exposure limit for radiation worker was set at 20 mSv , how many days can the worker report to the plant assuming they are limited to 8 -hour shift per day? Radiation weighing factor for x -rays $=1$

## Q25. NUCLEAR SAFETY (10 pts)

The Three Mile Island (TMI) accident in 1979 is the worst commercial nuclear power accident to have occurred in the United States. Although assessments have found that the accident did not result in any fatalities, it had a negative impact on the nuclear industry by increasing public fear and uncertainty regarding nuclear energy. For this reason, TMI is often attributed with slowing the growth of the nuclear industry in the U.S. The accident led to improvements in commercial nuclear reactor safety and regulation.

## TMI-2 Summary:

Type: Pressurized Water Reactor (PWR)
Rated Power: 2,722 MW-thermal, 959 MW-electric
Layout: two-loop with once-through steam generators

### 1.1 The TMI-2 reactor was operating at $97 \%$ of its rated power, when a failure of a pump caused the reactor to shutdown around 4:00 am on March 28, 1979. Due to a stuck pressure relief valve and operator error, water cooling to the core was insufficient to remove decay heat. <br> It is estimated that the reactor core was no longer covered with water by 6:00 am. What was the decay heat of the core in MW at this time? Use the following Wigner-Way decay heat formula and assume that the reactor had been operating at steady state for 30 days prior to the accident:

$$
P_{1}(t)=0.0622 P_{0}\left[t_{1}^{-0.2}-\left(t_{0}+t_{1}\right)^{-0.2}\right]
$$

where,
$P_{1}(t)=$ Thermal Power at time $t$
$P_{0}=$ Thermal Power before shutdown
$t_{0}=$ Time that the reactor has been operating before shutdown (seconds)
$t_{1}=$ Time since reactor shutdown (seconds)
1.2 Due to the lack of core cooling, the fuel in the TMI-2 core melted, resulting in failure of the fuel cladding and the release of radioactive material from the fuel. Therefore, during the accident, one of the primary physical barriers to the release of radioactive material was breached (fuel cladding). What are the two other primary physical barriers found in
typical commercial nuclear power plant designs that prevent the release of radioactive material to the environment?
1.3 The TMI-2 accident resulted in a release of radioactive material to the environment, as radioactive noble gases from the melted fuel escaped through the stuck open valve in the primary system and entered an auxiliary building at the reactor site. The largest release of radioactive material to the environment was Xe-133, which releases a beta particle and an 81 keV gamma ray when it undergoes radioactive decay.

The beta particle released from $\mathrm{Xe}-133$ is typically ignored due to its short average travel distance in air; however, the released gamma ray is a primary health concern. If the Xe -133 linear attenuation coefficient is $29.305 \mathrm{~cm}^{-1}$ for lead, what thickness of lead would be necessary to reduce the external exposure rate of $\mathrm{Xe}-133$ by 1,000 ?
1.4 It is estimated that the population living near the TMI plant may have received 1 mSv of ionizing radiation dose from the accident.

For simplicity, if it is assumed that the resulting dose was completely a result of external exposure to $\mathrm{Xe}-133$, what amount (mass) of Xe -133 would be necessary to result in 1 mSv ? Ignore contribution from the beta particle and utilize a gamma constant of $2.78 \mathrm{E}-5 \mathrm{mSv} / \mathrm{hr}$ per MBq and specific activity of $1.89 \mathrm{E} 5 \mathrm{Ci} / \mathrm{g}$. Assume the exposure occurred uniformly over 24 hours.
1.5 While population doses associated with the TMI-2 accidents are 0.5 pts estimated to be small, nuclear power plants have emergency plans in place to respond to potential accidents that result in the release of radioactive material to the environment. The population near a nuclear power plant may be ordered to take protective actions that can reduce their dose, such as evacuation. How does evacuation correspond to

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aspects of the radiation protection principles of time, distance, and shielding?
1.6 The guiding principle of 'ALARA' (As Low as Reasonably Achievable) serves as the guiding principle for radiation safety. ALARA means avoiding exposure to radiation that does not provide direct benefits, even if the dose is minimal. In the following questions, a group of workers are responsible for cleaning up TMI-2 following the accident, which involves the presence of radioactive water and may cause adverse effects towards the workers.

Given the dose rate generated by a source of $\mathrm{Cs}-137$ at 35 cm is $8.7 \mathrm{mGy} / \mathrm{h}$, determine the dose rate of the same source at 3 meters.
1.7 The dose rate of a source of Cs - 137 for the worker without shielding is calculated to be $12.5 \mathrm{mSv} / \mathrm{h}$. If the half value thickness of lead for Cs137 is 12.5 cm , calculate the thickness of lead required to reduce the initial dose level to be less than $10 \mu \mathrm{~Sv} / \mathrm{h}$. At the end, briefly explain the need to reduce the dose rate to less than $10 \mu \mathrm{~Sv} / \mathrm{h}$.
1.8 After the initial clean-up effort at TMI-2, approximately 400,000 gallons of radioactive water remained in the basement of the containment building. The principal sources of this radioactivity were Cs-137 at 156 $\mu \mathrm{Ci} / \mathrm{cm}^{3}$ and $\mathrm{Cs}-134$ at $26 \mu \mathrm{Ci} / \mathrm{cm}^{3}$. How many atoms per $\mathrm{cm}^{3}$ of these radionuclides were in the water at that time? The half-life of $\mathrm{Cs}-137$ is 30.17 years, while the half-life of Cs - 134 is 2.06 years.
1.9 The clean-up team at TMI-2 is composed of workers with varying levels of expertise in radiation safety. The individual dose rates from Cs-137 and Cs -134 for these workers are provided below:

Worker A: Cs-137 = $10 \mathrm{mSv} / \mathrm{h}, \mathrm{Cs}-134=5 \mathrm{mSv} / \mathrm{h}$
Worker B: Cs-137 $=8 \mathrm{mSv} / \mathrm{h}, \mathrm{Cs}-134=4 \mathrm{mSv} / \mathrm{h}$
Worker C: Cs-137 $=12 \mathrm{mSv} / \mathrm{h}, \mathrm{Cs}-134=6 \mathrm{mSv} / \mathrm{h}$

Assuming each worker is exposed for a total of 40 hours during the clean-up process, calculate the total dose received for each worker.
1.10 As part of the ongoing radiation safety assessment during the extended 1.0 pt clean-up at TMI-2, consider a mixture of radionuclides in the remaining water. Using the radioactivity values from (1.8) for Cs -137 and Cs -134 respectively, calculate the combined activity of both isotopes after 20 years. Science Olympiad

## QUESTION-SYLLABUS MATRIX

| Questions | Syllabus Topics |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1. Creation of new particles | ■ | $\square$ |  |  |  |  | $\square$ |
| 2. Criticality of a reactor |  |  | $\square$ |  |  |  |  |
| 3. Energy loss of neutron |  |  | $\square$ |  |  |  |  |
| 4. Mass of neutron star | ■ |  | $\square$ |  |  |  |  |
| 5. Radioactive dating |  | $\square$ |  | ■ |  |  |  |
| 6. Radioactive equilibrium |  | $\square$ |  |  |  |  |  |
| 7. Radioactivity as a source of heat |  | $\square$ | ■ |  |  |  | $\square$ |
| 8. Model of the atom | ■ |  |  |  |  |  |  |
| 9. Radioactive decay of H-3 |  | $\square$ |  |  |  |  |  |
| 10. Radioactive power systems |  | $\square$ | ■ |  |  |  | $\square$ |
| 11. Radioisotope application |  | $\square$ |  |  |  |  | $\square$ |
| 12. Nuclear binding energy | ■ |  | ■ |  |  |  |  |
| 13. Radiation detection |  | $\square$ |  |  |  |  |  |
| 14. Photon intensity |  | $\square$ |  |  |  |  |  |
| 15. Nuclear fission |  |  | $\square$ |  |  |  |  |
| 16. Fission product (Sm-157) |  | $\square$ | $\square$ |  |  |  |  |
| 17. $\mathrm{BF}_{3}$ neutron detection |  | $\square$ | $\square$ |  |  |  |  |
| 18. ALARA principle |  | $\square$ |  | ■ |  | $\square$ | $\square$ |
| 19. Positron Emission Tomography |  | $\square$ |  |  |  |  | $\square$ |
| 20. Half-life determination |  | $\square$ |  |  |  |  |  |
| 21. Fusion reaction | ■ | $\square$ | ■ |  |  |  |  |
| 22. Smoke detectors | ■ | $\square$ |  |  |  | ■ | $\square$ |
| 23. Radioactive dating |  | $\square$ |  | ■ |  |  |  |
| 24. Electron beam application |  | $\square$ |  |  |  | $\square$ | $\square$ |
| 25. Nuclear safety |  | $\square$ |  | $\square$ | $\square$ | $\square$ |  |

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## QUESTION FEEDBACK

Send your feedback on the questions through the link or QR code below:

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# See you at the <br> First International Nuclear Science Olympiad in the Philippines! 



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